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A MATHEMATICAL MODEL OF CHAOTIC PROCESSES IN A BOOST CONVERTER: GENERATION AND SYNCHRONIZATION

Yuval Beck[1] and Yefim Berkovich*[2]

[1]School of Electrical Engineering, Tel Aviv University (TAU), P.O. Box 39040, Tel Aviv 69978, Israel.

[2]Department of Electrical Engineering, Holon Institute of Technology, 52 Golomb st., Holon, 58102, Israel.

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***Corresponding Author**
Yefim Berkovich
Department of Electrical Engineering, Holon Institute of Technology, 52 Golomb st., Holon, 58102, Israel.

ABSTRACT

The paper presents a mathematical model, developed with a Matlab program, of a Boost Converter with a control system for studies of chaotic processes originating in the converter, and of various methods of preventing them. The choice of a mathematical model instead of known specialized programs for modeling power electronics circuits is due to the fact that such approach makes it possible to use the entire mathematical apparatus available for carrying out many calculations and processing their results. As will be seen from the paper, the model is sufficiently flexible and compact and allows for simple cyclic calculations of various chaotic processes. For the first time, the paper considers two ways of realization of Current Mode Control (CMC) – by the upper limit of pulses of the input inductance current, and by the lower one. Basing on these, the newly proposed ways of synchronization (preventing) of chaotic processes have been tested on the two mentioned control modes. In the conclusion, a number of calculations on the mathematical model have been compared with the results of modeling with the Orcad-Pspice program, which have shown their totally adequate character.

KEYWORDS: Chaos, Bifurcations, Synchronization, DC-DC Converters.

1. INTRODUCTION

In the recent decades, a multitude of studies and publications have been devoted to chaotic processes originating in non-linear dynamic systems of most varied nature. The results of these studies became new scientific fields – non-linear dynamics, synergetics, and a generalized information theory (Brillouin, 1963, Haken, 1983, Prigogine et al., 1984, 1994, Gleick, 1988, Doyle, 2011). A new paradigm of the basics of the course of all physical processes: mass-energy-information has come into being. Due to this, of special interest become simplified physical models, in particular, models based on electric circuits, which make it possible to generate chaotic processes, to study their course, and to determine their laws.

An example of such simple models is the boost converter, which transforms a certain DC voltage into a DC voltage at a higher level. Many studies dealing with the boost converter as a model for obtaining chaotic processes have been conducted, among which on should first and foremost to note (Deane, et al., 1990, 1992, Hamill, et al., 1992). These studies not only pay attention to the emergence of chaotic processes in boost converters in the CMC, but also give methodical bases for their studies.

A significant contribution to the studies of chaotic processes in various converter types used in power electronics has been made by C.K. Tse (Tse, 2003, 2004, Lu et al., 2000). These publications give the results of his studies of the generation of chaotic processes, the emergence of their various stages, and possibilities of synchronization. In order to describe these processes a vast mathematical apparatus of non-linear dynamics has been used – the bifurcation theory, Poincare's phase planes and maps, the Lyapunov exponent, Hopf's bifurcations, etc. The processes in different types of converters have been studied.

For estimating chaotic processes for a CMC boost converter bifurcation and statistical analysis are applied (Baranovski, et al, 1999, Woywode, et al., 2003) . In (Baranovski, et al, 2000) put stress on using the Fourier transformation and the power density spectrum (PDS), and the PDS of the inductor current of a boost converter has been calculated as an example. A specific feature of chaotic processes is that they decrease EMI in a system. Papers (Li, 2009) are dealing with this phenomenon, and (Baranovski, et al, 2003) gives a method of constructing a map of a chaotic process to be used for its further generation by a special control circuit. In (Zhang, et al., 2015) one finds a detailed consideration of this problem. The paper gives also other methods of spectral analysis for studying chaotic processes, namely the wavelet method and the Prony method.

Paper (Negoitescu, et al., 2008) investigates the bifurcation and chaotic behavior of a CMC Buck-Boost converter operating in continuous current mode (CCM). Its results have been obtained and processed using the following programs: CASPOS, Mathematica and Matlab. Similar processes in a buck converter with a closed loop control system have been considered in (Yuan, et al., 1998), and for PFC, in (Ghosh, et al., 2013). Paper (Zhioua, et al., 2014) is devoted to the application of the programs Matlab and PSIM for obtaining and study chaotic processes in a boost converter.

The influence of the output capacitor of the boost converter on the bifurcation periods is considered in (Dongale, 2015), where it is shown that they are increasing with the increase of the capacitor. Paper (Dongale, 2013) considers the influence of the input voltage and the load's resistivity on these processes. The dependence of the onset of bifurcations on the waveform of the current setting the CMC – from a constant value to various parameters of sawtooth waves – is studied in (Zafrani, et al., 1995). Synchronization possibilities are dealt with in (Bao, et al., 2008), where a strategy for excluding chaotic processes has been given.
It should be noted that while many papers give rather detailed descriptions of current modes, they give little space to the models themselves and the calculations based on them, and therefore there arises a need for a repeated development of a model, and a repeated compilation of programs for processing of the results.

The present paper is dealing with the creation and description of a most simple mathematical model which makes it possible to generate and study general laws of the course of chaotic processes. The model contains both a program for obtaining the necessary modes as well as programs for their mathematical processing. To that, Section 2 of the paper is devoted. Also in Section 2 and in Section 4 two chaotic processes are presented and described, which fundamentally differ in their directions. Sections 3 and 5 proposes a new method of synchronization (prevention) of chaotic processes, which has been shown to be efficient.

2. Mathematical model in the CMC-above mode

The presented in this paper a mathematical model is intended for studies of dynamic modes in a conventional DC-DC boost converter. If the boost converter is controlled by the Voltage Mode Control (VMC), then used the control system based on PWM, providing regulation of the duty cycle by varying the control voltage. Boost converter can be controlled by the CMC, and it is this method will be discussed below, since its use may arise the bifurcation and chaotic regimes, which are of interest to us.

As is known, this method is realized through comparing some value of current, I_{ref} with the current i_{in} of the inductance L_{in} of the converter (Fig. 1*a*), and provided that the condition $i_{in}=I_{ref}$ is fulfilled, the switch S opens, the current i_{in} decreases until the clock pulse comes, after that the switch S closes, the current i_{in} again increases until it reaches the current value I_{ref} and the process repeats itself. For the fixation of these two states an RS-trigger is used.

This control process where all the value of interest are denoted is shown in Fig. 2*a*. Fig. 2*b* shows another approach to realizing CMC: the value of the current I_{ref} is always less than i_{in}, if it decreases, and the condition $i_{in}=I_{ref}$ fulfils, the switch S closes, the current i_{in} increases until the clock pulse comes, when the switch S opens, and the current i_{in} decreases until it reaches the boost converter current value. The first method of current control (Fig. 1*a*, Fig. 2*a*) we shall call CMC-above, the second (Fig. 1*b*, Fig. 2*b*), - CMC-bottom.

The VMC does not lead to chaotic processes and is the most applicable in praxis. Note that if the duty cycle D changes, $V_o = V_{in}/(1-D)$, $I_{in} = V_{in}/R_o(1-D)^2$.Taking into account, that $\Delta i_{in} = V_{in} DT_s / L_{in}$, we get $I_{max} = I_{in} + \Delta i_{in}/2$ and $I_{min} = I_{in} - \Delta i_{in}/2$.

The dynamic description of the boost converter behaviour is set by a system of differential equations with respect to the current i_{in} and the voltage v_o:

$$\begin{aligned} L_{in} di_{in}/dt + d_1 \cdot v_o &= V_{in}, \\ C_o dv_o/dt + v_o/R_o &= d_1 \cdot i_{in}. \end{aligned} \qquad (1)$$

Here $d_1 = 1-d$ and d - is the commutation function, which equals *1* when the switch *S* is closed, and *0*, when it is open.

System (1) is not stiff, therefore let us substitute the differentials of the variables by their finite increments $\Delta i_{in} = i_{in}(\Delta t \cdot (k+1)) - i_{in}(\Delta t \cdot k)$, $\Delta v_o = v_o(\Delta t \cdot (k+1)) - v_o(\Delta t \cdot k)$

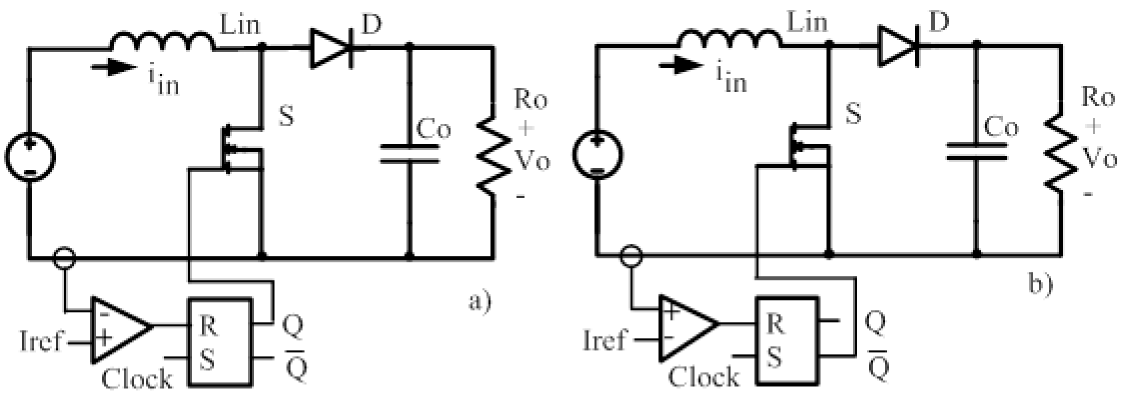

Fig. 1: Boost converter with CMC (a) CMC-above, (b) CMC-bottom.

for sufficiently small time intervals Δt on each k^{th} segment of calculation and change the form in which equations (1) are written:

$$\begin{aligned} i_{in}(\Delta t\cdot(k+1)) &= \left(V_{in}/L_{in} - d_1(\Delta t\cdot k)\cdot v_o(\Delta t\cdot k)/L_{in}\right)\cdot\Delta t + i_{in}(\Delta t\cdot k), \\ v_o(\Delta t\cdot(k+1)) &= \left(d_1(\Delta t\cdot k)\cdot i_{in}(\Delta t\cdot k)/C_o - v_o(\Delta t\cdot k)/R_oC_o\right)\cdot\Delta t + v_o(\Delta t\cdot k) \end{aligned} \quad (2)$$

or, by passing to the form of writing the variables i_{in} и v_o in the Matlab program, we get (instead of the "*" sign, the sign "$\cdot$" is used):

$$\begin{aligned} i_{in}(k+1) &= \left(V_{in}/L_{in} - d_1(k)\cdot v_o(k)/L_{in}\right)\cdot\Delta t + i_{in}(k), \\ v_o(k+1) &= \left(d_1(k)\cdot i_{in}(k)/C_o - v_o(k)/R_oC_o\right)\cdot\Delta t + v_o(k) \end{aligned} \quad (3)$$

Now write the equations that describe the functioning of the RS-trigger (Fig. 1*a*), which respectively form the variable d_1 in (3). For the trigger output *Q*, this will be a function $T_2(k)$ (or *d*), and for a free output $\overline{Q}$ - $T_1(k)$ (or 1-$T_2(k)$ $\rightarrow$ d_1). The outputs are complimentary to each other. First, we write the equation for the clock pulse. To do that, we form the equation of the sawtooth voltage $V_{ramp}(t) = t/T_s - floor\left(t/T_s\right)$ and by comparing this sawtooth voltage with the constant voltage V, we get a sequence of clock pulses. Here we mean that for discrete variables *k* and for $Vc(k) = V - V_{ramp}(k)$, and when $Vc(k) > 0$, the function $(Vc(k) > 0) = 1$, otherwise, it equals zero. The same holds for other expressions of the same form.

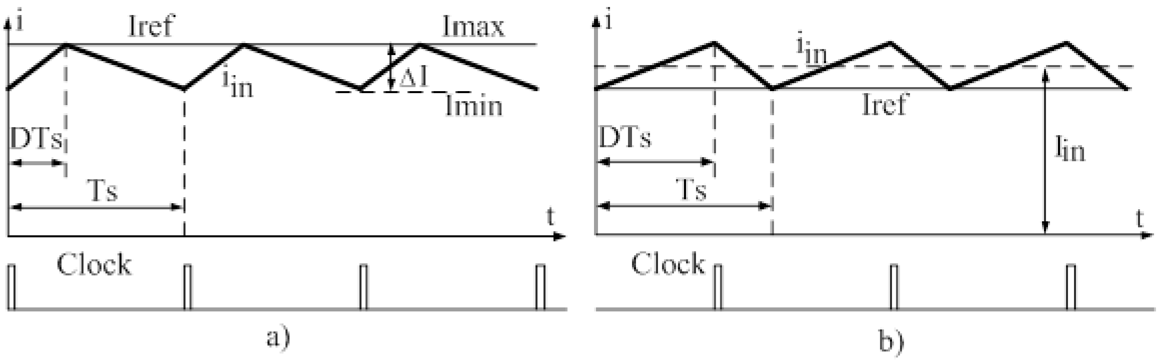

Fig. 2: Currents diagrams for CMC (a) CMC-above, (b) CMC-bottom.

With the arrival of the clock pulse $Vc(k)$ the output $\bar{Q}$ of the trigger equals zero, and when the current i_{in} reaches the value I_{ref}, the output equals one. Using logical functions, this can be written as follows:

$$T_1(k+1) = 1-(((V_c(k)>0)+T_2(k)>0))>0). \quad (4)$$

Indeed, immediately after passing a short clock pulse $(Vc(k)>0)=0$, $T_2(k)=1$, the expression in parentheses in (4) is equal *1* and $T_1(k+1)=0$.

For the output Q of the trigger, we have:

$$T_2(k+1) = 1-((T_1(k)>0)+(1>(I_{ref}>i_{in}(k)))>0). \quad (5)$$

Indeed, immediately after the current $i_{in}(k)$ reaches the current I_{ref} $(I_{ref}>i_{in}(k))=1$, further $(1>(I_{ref}>i_{in}(k)))>0)=0$, $T_1(k)=1$, the expression in parentheses in (5) is equal *1* and $T_2(k+1)=0$.

Let us introduce the reference values of the voltage and current as $V_B = V_{in}$, $I_B = V_{in}/R_o$ respectively. Accounting for this and taking into account (4) and (5), the complete system of equations describing the functioning of the boost converter in the CMC-above mode, takes on the form:

$$x1(1)=0; x2(1)=0; T_1(1)=1; T_2(1)=0; \iota_{ref}=3;$$
$$for\ k=1:10^5;$$
$$x1(k+1)=\left(\tau 1-\tau 1\cdot(1-T_2(k))\cdot x2(k)\right)\cdot\Delta\tau + x1(k);$$
$$x2(k+1)=\left(\tau 2\cdot(1-T_2(k))\cdot x1(k)-\tau 2\cdot x2(k)\right)\cdot\Delta\tau + x2(k); \quad (6)$$
$$T_1(k+1)=1-(((\upsilon(k)>0)+T_2(k)>0))>0);$$
$$T_2(k+1)=1-((T_1(k)>0)+(1>(\iota_{ref}>x1(k)))>0);$$
$$end$$

Here $x1(k) = i_{in}(k)/I_B$, $x2(k) = v_o(k)/V_B$, $\tau 1 = R_o T_s / L_{in}$, $\tau 2 = T_s / R_o C_o$,

$\iota_{ref} = I_{ref} / I_B$ $\upsilon(k) = V_c(k)/V_{in}$, $\Delta\tau = 0.5 \cdot 10^{-3}$, $T_s = 20 \cdot 10^{-6} s$.

The system of equations (6) serves as a basis of the mathematical model of the boost converter in the CMC-above mode. Figs. 3*a* and *b* gives the results of calculations in the Matlab program of the relative value of the current $x1(k)$ and of the relative value of the voltage $x2(k)$ for two values of ι_{ref} : $\iota_{ref} = 3.0$ and $\iota_{ref} = 4.8$. Here the values of the constants $\tau 1 = 4$, $\tau 2 = 0.1$, $\nu = V / V_B = 0.01$. In the first case, when the duty cycle $D \le 0.5$, the converter functions in a normal mode with the period T_s . In the second case, $D > 0.5$ and the converter move to a double-period mode, the period of work becomes $2T_s$, and the lower limit values of the current during a period acquire two different values. For comparison, these same regimes simulated in Pspice, which gave a complete coincidence and validation of the proposed model.

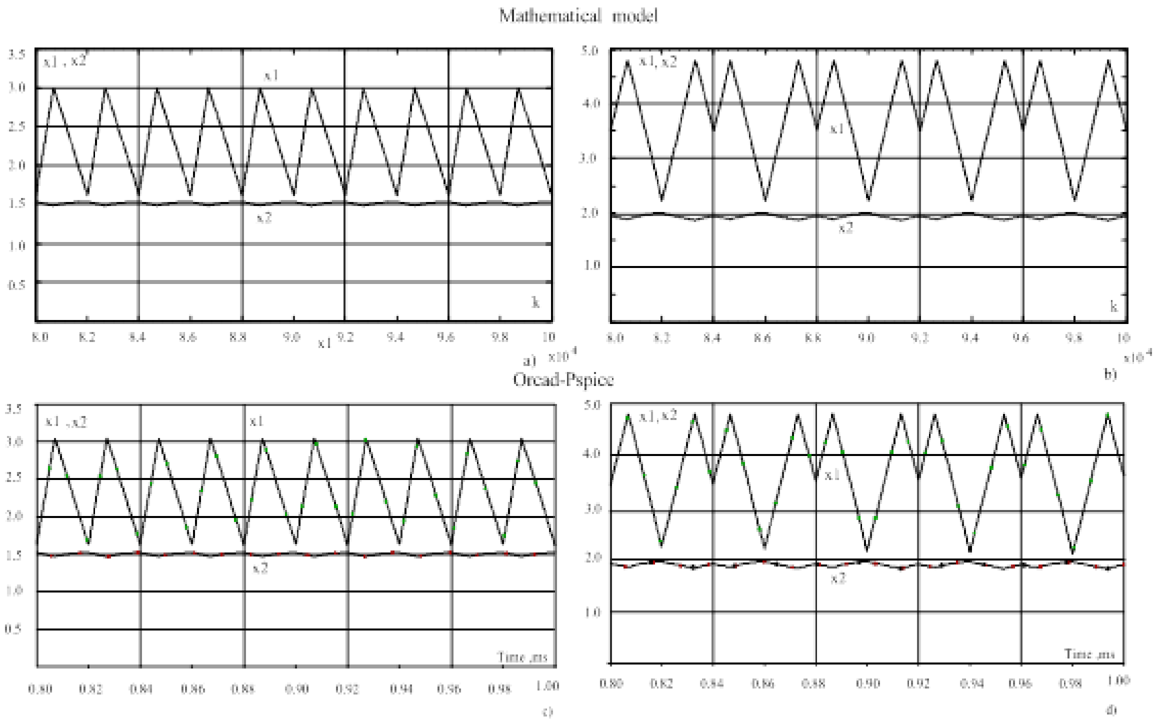

Fig. 3: Examples of output voltages and input currents in boost converter with CMC-above (a) i_{ref}=3 mode with period T_s, (b) i_{ref}=4.8 mode with period $2T_s$, (a) and (b) – calculated with the proposed model, (c) and (d) – calculated with Orcad-Pspice.

The system of equations (6), consisting of two equations for the variables *x1* and *x*2 and two more logical equations for the functioning of the RS trigger, completely describes all the processes in boost converter in CMC mode and allows you to get all the necessary dependencies and graphs. So, for example, in Fig. 4*a* shows the output signals *T1 (k)* and *T2 (k)* during the operation of the trigger in the steady-state period doubling mode ($\iota_{ref} = 5$), in Fig. 4*b* - the curve of the limiting cycle of the current ripple through the inductance *L* - $\tilde{x}1$ and its derivative - the voltage across the inductance *L* $\dot{\tilde{x}}1$.The next two graphs show the harmonic composition of these quantities, and graph (*e*) shows the phase plane of the quantities $\tilde{x}1$ and $\dot{\tilde{x}}1$ in chaos mode ($\iota_{ref} = 13.8$).

As is well known, after the period doubling mode a further increase of the reference current ι_{ref} will lead to the quadrupling of the period with four values of the lower limit current, then to octupling, etc., with the eventual setting of a chaotic process, which is characteristic by the absence of periodicity, and random changes of the lower limit values of the current.

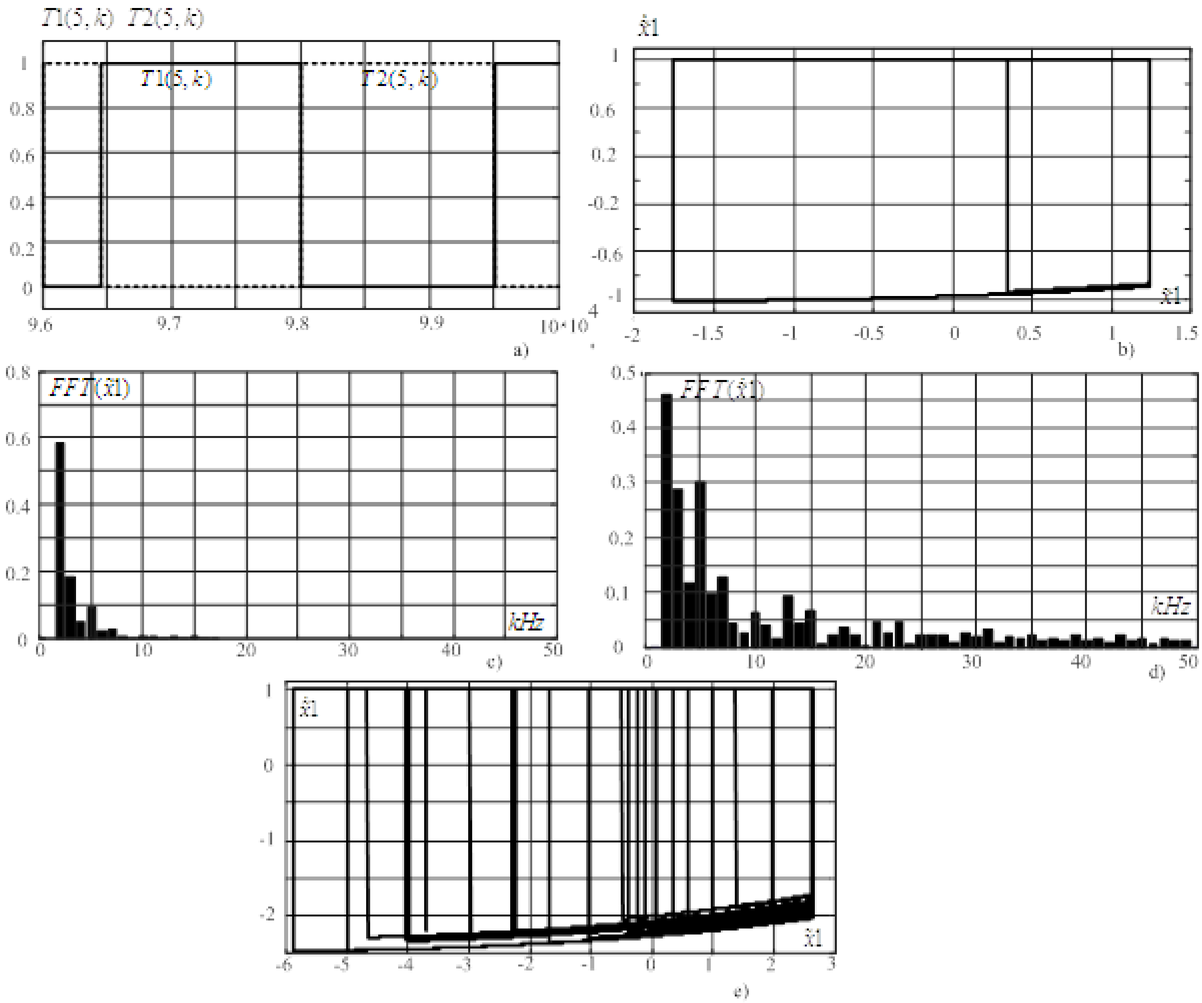

Fig. 4: Functional graphs of boost converter in CMC mode in steady state at $\iota_{ref} = 5$ (a) - output signals of the RS trigger, (b) - limit cycle in the plane $\tilde{x}1$- $\dot{\tilde{x}}1$, (c), (d) - harmonic

composition of current ripples $\tilde{x}1$ and voltage $\dot{\tilde{x}}1$, (e) - phase plane of the quantities in chaos mode (ι_{ref} = 13.8).

In order to obtain all these modes, we add to system (6) an equation describing changes of $\iota_{ref}(j) = 3.0 + 0.02j$, and carry out cyclic calculations of modes for different *j* while changing *k* until a steady state is reached in each case. In this case, the mathematical model assumes the form (7).

Carrying out calculations by the Matlab program, we get the values of the currents $x1(k)$ and the voltages $x2(k)$ for all the modes of functioning of the boost converter, from a periodic one with the period T_s, and until the chaotic mode sets on.

$$\begin{array}{l}
for\ \ j = 1:500; \\
\quad for\ \ k = 1:10^5; \\
\iota_{ref}(j) = 3.0 + 0.02 \cdot j; \\
x1(j,k) = 0; x2(j,k) = 0; T_1(j,k) = 1; T_2(j,k) = 0; \iota_{ref}(1) = 3; \\
x1(j,k+1) = \left(\tau 1 - \tau 1 \cdot (1 - T_2(j,k)) \cdot x2(j,k)\right) \cdot \Delta\tau + x1(j,k); \\
x2(j,k+1) = \left(\tau 2 \cdot (1 - T_2(j,k)) \cdot x1(j,k) - \tau 2 \cdot x2(j,k)\right) \cdot \Delta\tau + x2(j,k); \\
T_1(j,k+1) = 1 - (((\upsilon(k) > 0) + (T_2(j,k) > 0)) > 0); \\
T_2(j,k+1) = 1 - ((T_1(j,k) > 0) + (1 > (\iota_{ref}(j) > x1(j,k))) > 0); \\
\quad end \\
end
\end{array} \quad (7)$$

As was already noted, for the mode with the period T_s, we obtain one value of the current at the lower limit, for the doubling of the period, two values, for quadrupling, four, et etc., and for the chaotic mode, the number of these values increases infinitely.

This picture is usually represented with a plot of the dependence of the lower limit values on the current ι_{ref} changes - Fig. 5 shows the results of these calculations, obtained with a simple program:

$$\begin{array}{l}
j = 1:500; \\
m = 10^5 - [1:25] \cdot 2000; \\
w1 = x1(j,m); \\
plot(3 + 0.02 \cdot j, w(:,:), 'k.');
\end{array} \quad (8)$$

It is seen that the first mode materializes up to the value $\iota_{ref} = 4.5$, the second mode – with the period doubling of the period – up to the value $\iota_{ref} = 6$, further follows the period quadrupling mode, then to octupling, which quickly transforms into the chaotic mode with respect to the values of the lower limits of the currents.

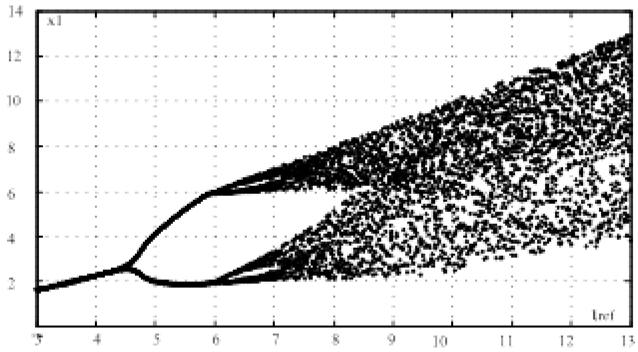

Fig. 5: Plot bifurcation diagrams for different values of ι_{ref} (CMC-above).

The curve in Fig. 5 is a typical one for determined chaos modes, independently of the nature of phenomena, whether for the process of procreation of a population described by the logistical equation (Gleick, 1988), or, as in our case, for changing of modes in a boost converter controlled by the CMC- above method.

As can be seen from the above material, the magnitude of the current ripple $\tilde{x}1$ through the inductance *L* significantly affect the nature of the mode in boost converter. These ripples, together with the voltage $\dot{\tilde{x}}1$ across the inductance, determine the value of reactive (non-active) power, circulating between the input source and the converter. It can be shown that the area of the limit cycle in the plane $\tilde{x}1$ - $\dot{\tilde{x}}1$ (for example, Fig. 4*b*), divided by 2π, gives the value of this power, calculated by the formula

$$Q = \sum_{i=1}^{\infty} i \cdot V_i \cdot \tilde{I}_i \cdot \sin\varphi_i \,, \qquad (8a)$$

where $V_i, \tilde{I}_i$ - are the effective values of the voltage and current ripple, φ_i is the phase angle between them, *i* - is the harmonic number of these quantities (Emde, 1921, Mayevsky, 1978). In this particular example Fig. 4*b*, reactive power, calculated through the area, *Q =1.21*. The same power, calculated by the formula, through the harmonic composition gives *Q =1.19*. As will be seen from subsequent sections, it is precisely the effect on current ripple and reactive power that makes it possible to avoid the bifurcation regime and chaos (see also (Beck, et al., 2019)).

3. Synchronization method in the CMC-above mode

Consider now the method proposed in this paper, of preventing chaotic modes independently of the value of duty cycle *D*. A schematic for realization of the method is given in Fig. 6*a*. As is seen from it, in parallel to the input inductance, a switch S1 is connected, which closes for a short period (approximately for the time $\approx 0.05T_s$) upon reaching the lower limit of the inductance current of some current ι_{ref1}, which approximately equals its value when functioning in a usual VMC mode. The current ι_{ref1} varies with the variations of the current ι_{ref}, that is, increases or decreases simultaneously with it. The boost converter control system is supplemented by yet another RS-trigger (Fig. 6*a*, RS1), which forms a pulse at the output $\bar{Q}1$ when the inductance current reaches the current ι_{ref1}, and changes its stet when clock pulse comes to the other input.

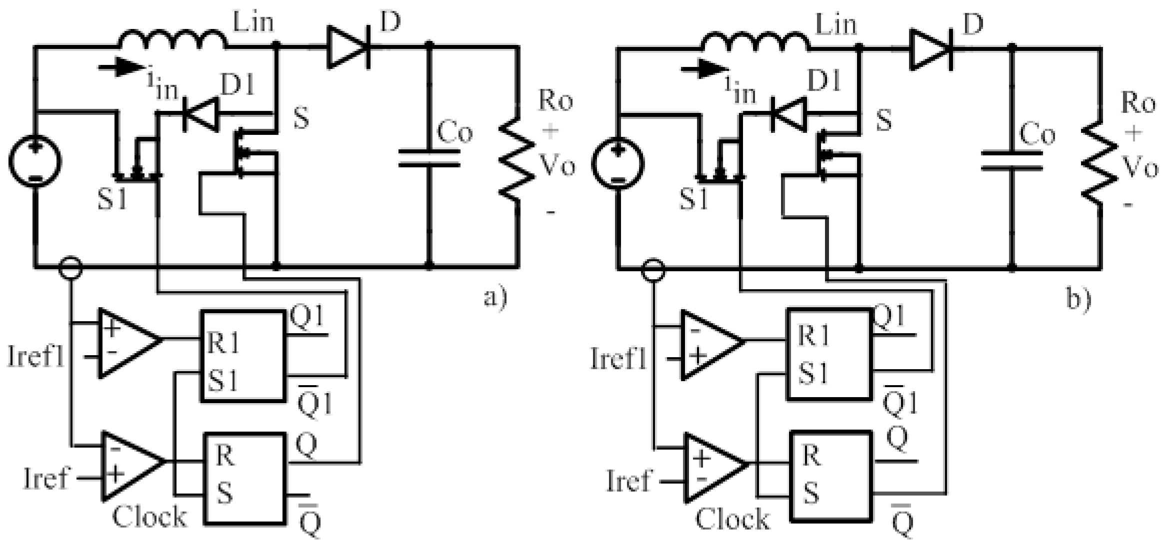

Fig. 6: Boost converter with synchronization of chaotic process (a) for CMC-above, (b) for CMC-bottom.

Taking into account the change made in the boost converter circuit, its mathematical model should be supplemented by equations describing the switching of the RS1-trigger, similar to those, which were introduced for the first trigger. On the basis of the conditions of the RS1 functioning, these equations assume the following form:

$$T_3(k+1)=1-(((T_4(k)>0)+(\iota_{ref}(j)>x1(k)))>0);$$
$$T_4(k+1)=1-(((x1(k)>\iota_{ref1}(j))+(T_3(k)>0)+(T_2(k)>0))>0) \quad (9)$$

Here, the function $T_3(k)$ reflects the *Q1* output operation, and the function $T_4(k)$ - a output $\bar{Q}1$ of the trigger. Now the system of equations, and respectively, the mathematical model describing the processes in the circuit in Fig. 6*a*, assumes the following form:

$$for\ \ j=1:500;$$
$$for\ \ k=1:10^5;$$
$$\iota_{ref}(j)=3.0+0.02\cdot j;\iota_{ref1}(j)=1.2+0.018\cdot j;$$
$$x1(1,k)=0;x2(1,k)=0;T_1(1,k)=1;T_2(1,k)=1;T_3(1,k)=0;T_4(1,k)=0;\ \iota_{ref}(1)=1.5;\iota_{ref1}(1)=0.8;$$
$$x1(j,k+1)=\left(\tau1\cdot(1-T4(j,k))-\tau1\cdot(1-T_2(j,k)-T4(j,k))\cdot x2(j,k)\right)\cdot\Delta\tau+x1(j,k);$$
$$x2(j,k+1)=\left(\tau2\cdot(1-T_2(j,k)-T4(j,k))\cdot x1(j,k)-\tau2\cdot x2(j,k)\right)\cdot\Delta\tau+x2(j,k);$$
$$T_1(j,k+1)=1-(((\upsilon(k)>0)+T_2(j,k)>0))>0);$$
$$T_2(j,k+1)=1-((T_1(j,k)>0)+(1>(\iota_{ref}(j)>x1(j,k)))>0); \quad (10)$$
$$T_3(j,k+1)=1-(((T_4(j,k)>0)+(\iota_{ref}(j)>x1(j,k)))>0);$$
$$T_4(j,k+1)=1-(((x1(j,k)>\iota_{ref1}(j))+(T_3(j,k)>0)+(T_2(j,k)>0))>0);$$
$$end$$
$$end$$

This system of equations, as is seen, is supplemented by the equation for the varying current ι_{ref1} and to the first and second equations is added function $T_4(j,k)$ with accounts for the switching of the latch RS1 of the current. The plot of variations of the lower values of the current is given in Fig. 7. Comparing the latter plot with the plot in Fig. 5, we see that no changes in the duty cycle $D>0.5$ are observed in a wide range, no current bifurcations are observed, the lower values of the current in the entire range equal the same value, and, as a result chaotic processes do not emerge. For the maximal value of the current ι_{ref} the values of the input current and the output voltage of the boost converter are, respectively, $mean(x1(500,:))=11.597$, $mean(x2(500,:))=3.395$ ($D=0.705$). The output power also increases, which previously was bounded in the CMC mode. Thus, the proposed method of removal of chaotic processes has shown its efficiency.

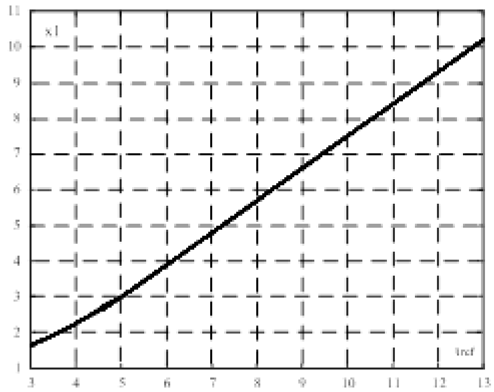

Fig. 7: Plot of variations of the lower limit values of the current x1 (iin) with synchronization of the CMC-above.

4. Description of the mathematical model in the CMC-bottom mode

Consider further the CMC mode when the switching of the main switch S occurs when the lower limit of the input current i_{in} reaches the current ι_{ref} (the CMC-bottom mode, Fig. 1*b*, Fig. 2*b*). The complete system of equations in relative units that describes the functioning of the boost converter in the CMC- button mode assumes the form (11):

$$\begin{aligned}
&x1(k+1)=\left(\tau 1-\tau 1\cdot(1-T_2(k))\cdot x2(k)\right)\cdot\Delta\tau+x1(k);\\
&x2(k+1)=\left(\tau 2\cdot(1-T_2(k))\cdot x1(k)-\tau 2x2(k)\right)\cdot\Delta\tau+x2(k);\\
&T_1(k+1)=1-(((1>(x1(k)>\iota_{ref}))+(T_2(k)>0))>0);\\
&T_2(k+1)=1-(((\upsilon(k)>0)+T_1(k)>0))>0);
\end{aligned}\qquad(11)$$

Here the first two equations remained unchanged, while two others describe the conduct of the RS–trigger in this control method: the arrival of the clock pulse opens the switch S (there is no pulse on the output of the trigger RS), and when the lower limit of the current i_{in} reaches the value ι_{ref} a pulse appears at that output, which closes the switch S. Figs. 8*a* and *b* gives the results of calculations in the Matlab program of the relative value of the current *x1(k)* and the relative value of the voltage *x2(k)* for two values of ι_{ref}=4.85 and ι_{ref}=2.5.

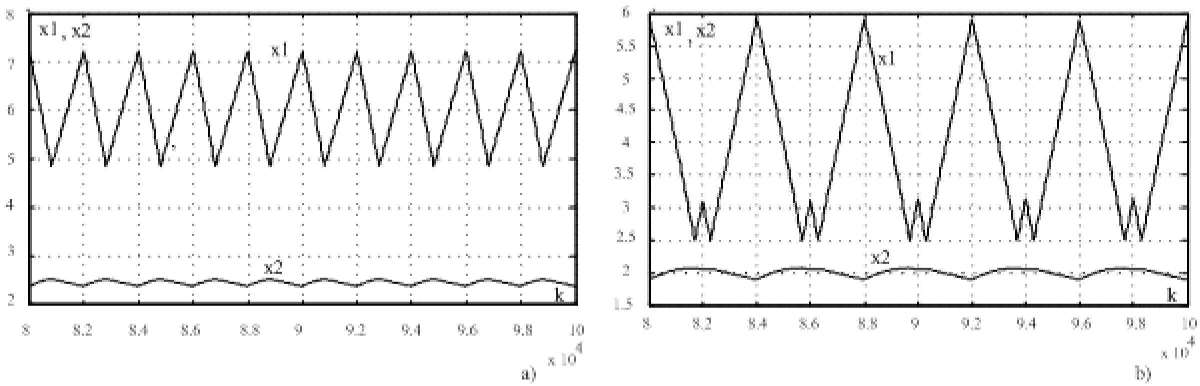

Fig. 8: Examples of output voltage and input current in boost converter with CMC-bottom mode (a) ι_{ref}=4.85, mode with period T_S, (b) ι_{ref}=2.5, mode with period $2T_S$.

Here the values of the constants $\tau 1$ and $\tau 2$, as well as the function $V_c(k)$ are the same as in the case of CMC- above. In the first case, when the duty cycle $D > 0.5$, unlike the CMC-above, the converter functions in the normal mode with the period T_s. In the second case, $D < 0.5$ and the converter passes to the mode of double period, the period of functioning becomes $2T_s$, and the upper limit current values acquire two different values during the period. The further lowering of the reference current ι_{ref} will lead to the quadrupling of the period, with four values of the upper limit current, etc., then octupling, finally resulting in the emergence of a chaotic process whose specific feature is the absence of periodicity and random changing of upper limit current values.

To obtain all these modes, we add to system (11) an equation of changes $\iota_{ref}(j) = 5.0 - 0.016j$, and carry out cyclical calculations of modes for various j when k changes until in each case a steady state mode establishes. In that case the mathematical model acquires the form (12). Completing the calculation in program, we obtain the values of the currents *x1(k)* and voltages *x2(k)* in all the modes of functioning of the boost converter, from the periodical one with the period T_S until the chaotic process establishes. As was already noted, in the mode with the period T_S we obtain one value of the current at the upper limit, upper the doubling of period, two values, upon quadrupling , four, etc., while in the chaotic mode there appears an infinite number of values. We represent this picture, as in the mode CMC-above, with a plot of the dependence of the upper limit current values on the variation of the current ι_{ref} (Fig. 9).

$$
\begin{aligned}
&\textit{for } \ j = 1:500;\\
&\quad \textit{for } \ k = 1:10^5;\\
&\iota_{ref}(j) = 5.0 - 0.008 \cdot j;\\
&x1(j,1) = 0; x2(j,1) = 0; T_1(j,1) = 1; T_2(j,1) = 0;\\
&x1(j,k+1) = \left(\tau 1 - \tau 1 \cdot (1 - T_2(j,k)) \cdot x2(j,k)\right) \cdot \Delta\tau + x1(j,k);\\
&x2(j,k+1) = \left(\tau 2 \cdot (1 - T_2(j,k)) \cdot x1(j,k) - \tau 2 \cdot x2(j,k)\right) \cdot \Delta\tau + x2(j,k);\\
&T_1(j,k+1) = 1 - (((1 > (x1(j,k) > \iota_{ref}(j))) + (T_2(j,k) > 0)) > 0);\\
&T_2(j,k+1) = 1 - (((\upsilon(k) > 0) + T_1(j,k) > 0)) > 0);\\
&\quad \textit{end}\\
&\textit{end}
\end{aligned}
\qquad (12)
$$

We see that the first mode is realized until the value $\iota_{ref} = 3.5$, the second mode of the doubling of the period, until the value $\iota_{ref} = 1.8$, further follows the mode of the quadrupling of the period (until the value $\iota_{ref} = 1.5$), which quickly transforms into the octupling mode, and further, into a chaotic mode with respect to the values of the lower limits of the currents. The curve in Fig. 9, although is a typical in the whole, but it is a metter of principle that in this case the process is directed downwards, and the chaotic mode emerges upon the decrease of the current.

5. Synchronization method in the CMC-bottom mode

Consider a method for preventing the emergence of chaotic processes independently of the value *D* of the duty cycle also for the CMC-bottom mode. The schematic for the realization

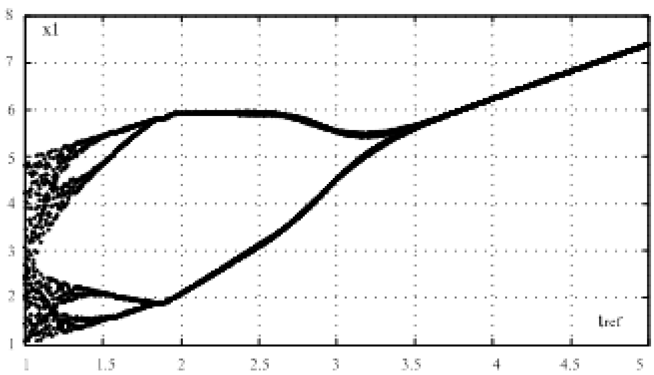

Fig. 9: Plot bifurcation diagrams for different values of ι_{ref} (CMC-bottom).

of this method is given in Fig. 6*b*. As in Fig. 6*a* , a switch is connected in parallel to the input inductance, which closes for a short time (approximately for $0.05T_s$) when in this case is reached the upper limit of some inductance current ι_{ref1}, more or less equal to its value upon functioning in the VMC regular mode. The control system of the boost converter in this case is similar to that described for Fig. 6*a*. On the basis if the conditions of functioning of the RS1, equations (13) are supplemented by the following equations:

$$T3(k+1) = 1 - (((x1(k) > i_{ref}(j)) + (T4(k) > 0)) > 0);$$
$$T4(k+1) = 1 - (((T3(k) > 0) + (T1(k) > 0) + (i_{ref1}(j) > x1(k))) > 0); \qquad (13)$$

Now the system of equations, and consequently, the mathematical model that describes the processes in the schematic in Fig. 6*b*, assumes the following form:

$$\begin{aligned}
&for\ j = 1:245;\\
&\quad for\ k = 1:10^5;\\
&\iota_{ref}(j) = 5 - 0.016 \cdot j;\ \ \iota_{ref1}(j) = 6.9 - 0.022 \cdot j;\\
&x1(j,1) = 0; x2(j,1) = 0; T1(j,1) = 1; T2(j,1) = 0; T3(j,1) = 0; T4(j,1) = 0;\\
&x1(j,k+1) = (\tau 1 \cdot (1 - T4(j,k)) - (x2(j,k) \cdot \tau 1) \cdot (1 - (T2(j,k) \,|\, T4(j,k))))dt + x1(j,k);\\
&x2(j,k+1) = ((x1(j,k) \cdot \tau 2) \cdot (1 - (T2(j,k) \,|\, T4(j,k))) - x2(j,k) \cdot \tau 2)dt + x2(j,k);\\
&T1(j,k+1) = 1 - (((1 > (x1(j,k) > \iota_{ref}(j))) + (T2(j,k) > 0)) > 0);\\
&T2(j,k+1) = 1 - (((\upsilon(k) > 0) + (T1(j,k) > 0)) > 0);\\
&T3(j,k+1) = 1 - (((x1(j,k) > \iota_{ref}1(j)) + (T4(j,k) > 0)) > 0);\\
&T4(j,k+1) = 1 - (((T3(j,k) > 0) + (T1(j,k) > 0) + (\iota_{ref1}(j) > x1(j,k))) > 0);\\
&\quad end\\
&end
\end{aligned} \qquad (14)$$

As is seen from this system of equations, it is supplemented by an equation for the current ι_{ref1}, and in the first and second equations the current function $T4(j,k)$, which accounts for the switching of the RS1- trigger. The plot of the upper limit values of the current calculated by this method is shown in Fig. 10. It is seen that that in this case the proposed method of preventing chaotic processes also showed its efficiency.

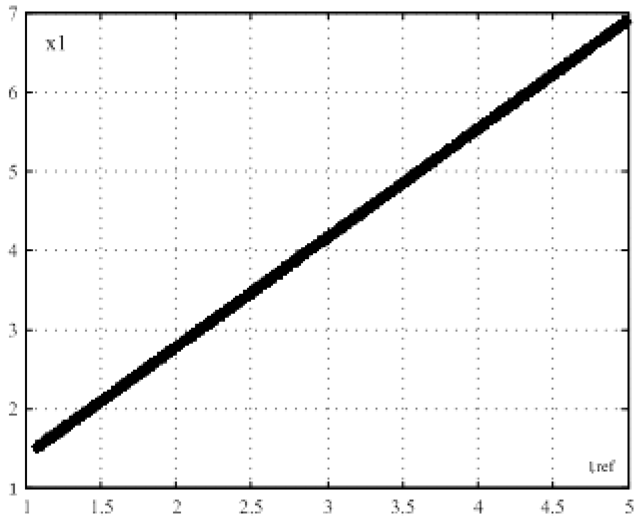

Fig. 10: Plot of variation of the upper limit values in the current x1(k) with synchronization in CMC-bottom.

6. CONCLUSIONS

1. A simple model of a boost converter in CMC has been developed on the basis of the Matlab platform, which makes it possible not only quickly to obtain all the modes of the functioning of the converter, including chaotic ones, but in this model itself, to carry out numeric and graphic processing of results.
2. Two control modes by currents, the CMC-above and CMC-bottom have been considered and studied, for which two kinds of differing in principle bifurcation diagrams are obtained, that is coming of chaos for increasing quantities in the first case, and for their decreasing, in the second case.
3. For each kind of controls considered, methods of synchronization of chaotic processes have been proposed that show their efficiency. In both cases, the methods are based on the effect on the ripple of the reactor current and, accordingly, on the reactive power of the current ripple circulating between the source and the converter.

REFERENCES

1. Bao, B-C., Xu, J-P., and Liang, Y. (2008), "Inductor Current Sampled Feedback Control of Chaos in Current-Mode Boost Converter", Journal of Electron. Science and Technology of China, 2008; 6(1): 52-55.
2. Baranovski, A. L., and Schwarz, W. "Statistical analysis and design of continuous-discrete chaos generators", IEICE Trans. Fundamentals, E82-A(9), September, 1999.

3. Baranovski, A. L., *et al*. (2000), "Chaotic control of a dc-dc converter", IEEE Int. Symposium on Circuits and Systems, ISCAS 2000, Switzerland, May 2000.
4. Baranovski, A. L., Schwarz, W. (2003), "Chaotic and random point processes: Analysis, design and applications to switching systems", IEEE Trans. on Circuits and Systems - August, 2003; 50: 1081-1088.
5. Beck, Y., Berkovich, Y. (2019), "Modes of bifurcations, chaos, and undamped oscillations in a buck converter with feedback and their synchronizing", Int. Journal of Computer and Electrical Engineering (IJCEE), December, 2019; 11(4): 164-174.
6. Brillouin, L. (1963), "Science and Information Theory" (2nd Ed., Chicago, IL, Academic Press, 1963.
7. Deane, J., and Hamill, D. (1990), "Instability, subharmonics and chaos in power electronic systems", IEEE Trans. on Power Electron., 1990; 5(3): 260–267.
8. Deane, J. H. B. (1992), "Chaos in current-mode controlled boost DC-DC converter", IEEE Trans. on Circuits and Systems-I, 1992; 39(8): 680-683.
9. Dongale, T. D. (2015), "Investigation of Nonlinear Dynamics in the Boost Converter: Effect of Capacitance Variations", Int. Journal on Computational Science & Applications (IJCSA), June 2015; 5(3): 67-76.
10. Dongale, T. D. (2013), "Simulative Study of Nonlinear Dynamics in Single Stage Boost Converter", Int. Journal of Chaos, Control, Modelling and Simulation (IJCCMS), September, 2013; 2(3): 59-65, 2013.
11. Emde, F. (1921), "Zur Definition der Scheinleistung und der Blindleistung bei ungleichmeassig belasten Mehrphasensystema", Elektrotech. Maschinenbau, Bd. 39, S., 1921.
12. Gleick, J. "Chaos. Making a new science", Abacus, 1988.
13. Ghosh, A., Saha, P., and Panda G. (2013), 'Chaos: A Nonlinear Phenomenon in AC-DC Power-Factor-Corrected Boost Convertor', Int. J. Com. Dig. Sys., 2013; 2(3): 167-172.
14. Haken, H. (1983), "Synergetics. An Introduction", Springer, Berlin, 1983.
15. Hamill, D., Deane, J., and Jefferies, D. (1992), "Modeling of Chaotic DC-DC Converters", IEEE Trans. on Power Electron., January, 1992; 1: 25-36,.
16. Li, H. (2009), "Reducing Electromagnetic Interference in DC-DC converters with Chaos Control", Dissertation, Hagen, 2009.
17. Lu, H. H. C., and Tse, C. K. (2000), "A Study of Synchronization in Chaotic Autonomous C´uk DC/DC Converters", IEEE Trans. on Circuits and Systems-I, June, 2000; 47(6): 913-918.

18. Mayevsky, O. (1978), "Energy efficiency of semiconductor converters", Energy, Moscow, 1978.
19. Negoitescu, D., Lascu, D., Popescu, V., *et al*, "Bifurcation and Chaotic Aspects in Peak Current Controlled Buck-Boost Converters", WSEAS Trans. on Circuits and Systems, 2008; 688-697.
20. Prigogine, I., Stengers, I. (1994), "The Time, Chaos and Quant", 1994.
21. Prigogine, I., Stengers, I. (1984), "Order out of Chaos", Heinemann, London, 1984.
22. Tse, C. K. "Chaos in Power Electronics", IEEE ISCAS'2003, Tutorial Notes, 2003; 28.
23. Tse, C. K. (2004), "Complex Behavior of Switching Power Converters", CRC Press., 2004.
24. Woywode, O., Weber, J., Güldner, H., *et al*. (2003), "Bifurcation and Statistical Analysis of DC–DC Converters", IEEE Trans. on Circuits and Systems, August, 2003; 50: 1072-1080.
25. Yuan, G., Banerjee, S., Ott, E., *et al*. "Border–Collision Bifurcations in the Buck Converter", IEEE Trans. on Circuits and Systems - I, 1998.
26. Zhang, B., and Wang, X. (2015), "Chaos Analysis and Chaotic EMI Suppression of DC-DC Converters", John Wiley & Sons Singapore Pte. Ltd., 2015.
27. Zafrani, I., Ben-Yankov, S. (1995), "A chaos model of subharmonic oscillations in current mode PWM boost converters", Proc. IEEE Power Electron. Special Conf. PESC, 1995.
28. Zhioua, M., Belghith, S. (2014), "Analysis of the Chaotic Discrete Model of Boost Converter", Int. Conf. on Automation, Control, Engineering and Computer Science (ACECS'14), 2014; 5-11.