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# AN M/M/C ENCOURAGED ARRIVAL FEEDBACK QUEUING MODEL WITH CUSTOMER IMPATIENCE

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## ABSTRACT

Understanding the behavior of the customers is the most challenging task which firms have to face in today's scenario of cut-throat competition. In order to gain competitive edge firms often introduce lucrative offers and discounts due to which the customers get attracted towards particular firm these customers are termed as *encouraged arrivals*. As a consequence of encouraged arrivals more and more customers join the system hereby increasing the time, a customer will have to spend in waiting for the service which results in customer impatience. These impatient customers may leave the system without service. Firms often deploy various strategies to retain these reneging customers for sustainable growth. Encouraged arrivals also put service facility under pressure due to which some customers, dissatisfied with the service join the queue again as *feedback customers* for satisfactory service. In this paper we develop a queuing model with all issues as discussed above and solve the model in steady state to derive various probabilistic and performance measures and we also present numerical illustration for better understanding of the applicability of the model. Having known these performance measures in advance various strategies can be designed for smooth administration and functioning of the system.

**KEYWORDS:** Encouraged arrivals, multi-server queuing model, feedback queue, reneging, retention, customer impatience.

## 1 INTRODUCTION

In today's scenario of fierce competition every firm strives for sustainable growth and in order to design sustainability strategies, understanding the functioning of the overall system and uncertain customers' behavior is of utmost importance & need. Easy access to global markets at their fingertips with smartphones, customers have enormous choices available to them and they choose the one offering them best deals and discounts. So to attract customers firms often introduce lucrative offers and announce sale. The customers who thus get attracted towards the firm are termed as *encouraged arrivals*. During the sale period firms often experience heavy rush and customers have to wait for longer in the queues to avail service. Some customers come with a threshold limit of time and having waited for longer time, they leave the system. Customers thus leaving can prove to be detrimental for the reputation of the firm and result in loss of revenue as well as goodwill. This can hamper the sustainable growth of the firm as customers with easy access to various social media platforms can post negative comments which can hamper the reputation and goodwill of the firm, so firms deploy various strategies to retain these customers to the best possible extent. Heavy rush at times puts limited service facility under pressure and that in turn can result in dissatisfaction among certain customers. Customers dissatisfied with the service join the queue again as *feedback customers* for satisfactory service. This paper is an extension of the work by authors in,[9] with infinite capacity system. As now-a-days with various online e-commerce platforms there is no restriction of physical space needed for customers, and innately infinite number of customers can join the system through various online e-commerce platforms. This elaborates the significance of studying infinite capacity model. Detailed literature can be seen in Som and Seth.[9]

Rest of the paper is organized as follows: Formulation of the model is presented in section 2. Steady-state probabilities of the models are derived recursively in section 3. Various performance measures are presented in section 4. Section 5 presents a real-life numerical illustration and conclusion and future scope is given in section 6.

## 2 Mathematical Model Formulation

Model is formulated with same assumptions as in Som and Seth,[9] with an exception that capacity of the system is considered to be innately infinite here.

Steady-State difference equations governing the model are given by:

$$0 = -\lambda(1+\eta)P_0 + \mu p P_1;\ n = 0 \qquad (1)$$

$$0 = \lambda(1+\eta)P_{n-1} - \{\lambda(1+\eta) + n\mu p\}P_n \ + \ (n+1)\mu p P_{n+1}; 1 \le n \le c-1 \qquad (2)$$

$$0 = \lambda(1+\eta)P_{n-1} - \{\lambda(1+\eta) + c\mu p + (n-c)\xi p'\}P_n \ + \{c\mu p + (n+1-c)\xi p'\}P_{n+1}; n \ge c \ (3)$$

**3 Steady-state solution**

On solving (1) - (3) recursively we get;

$$P_n = Pr\{'n' \, customers \, in \, the \, system\}$$

$$= \begin{cases} \dfrac{1}{n!}\left(\dfrac{\lambda(1+\eta)}{\mu p}\right)^n P_0; & 1 \le n \le c \\ \dfrac{1}{c!}\left(\dfrac{\lambda(1+\eta)}{\mu p}\right)^c \displaystyle\prod_{k=c+1}^{n}\left(\dfrac{\lambda(1+\eta)}{[c\mu p + (k-c)\xi p']}\right) P_0; & n > c \end{cases} \qquad (4)$$

Using condition of normality $\sum_{n=0}^{\infty} P_n = 1$, we get

$P_0$ = Pr{no customer in the system}

$$= \left[\sum_{n=0}^{c}\frac{1}{n!}\left(\frac{\lambda(1+\eta)}{\mu p}\right)^n + \frac{1}{c!}\left(\frac{\lambda(1+\eta)}{\mu p}\right)^c \sum_{n=c+1}^{\infty}\left\{\prod_{k=c+1}^{n}\left(\frac{\lambda(1+\eta)}{[c\mu p + (k-c)\xi p']}\right)\right\}\right]^{-1}$$

$$= \left[\sum_{n=0}^{c}\frac{1}{n!}\left(\frac{\lambda(1+\eta)}{\mu p}\right)^n + \frac{1}{c!}\left(\frac{\lambda(1+\eta)}{\mu p}\right)^c e^{\frac{\lambda(1+\eta)}{\xi p'}}\left(\frac{\lambda(1+\eta)}{\xi p'}\right)^{-\frac{c\mu p}{\xi p'}}\left[\Gamma\left(\frac{c\mu p + \xi p'}{\xi p'}\right) - \Gamma\left(\frac{c\mu p}{\xi p'} + 1, \frac{\lambda(1+\eta)}{\xi p'}\right)\right]\right]^{-1} \qquad (5)$$

**4 Measures of Performance**

1. **Probability that a customer has to wait ($P_b$):**

$$P_b = \Pr\{all \ servers \ are \ busy\}$$

$$= \sum_{n=c}^{\infty} P_n = \left[\frac{1}{c!}\left(\frac{\lambda(1+\eta)}{\mu p}\right)^c \left\{1 + e^{\frac{\lambda(1+\eta)}{\xi p'}}\left(\frac{\lambda(1+\eta)}{\xi p'}\right)^{-\frac{c\mu p}{\xi p'}}\left[\Gamma\left(\frac{c\mu p + \xi p'}{\xi p'}\right) - \Gamma\left(\frac{c\mu p}{\xi p'} + 1, \frac{\lambda(1+\eta)}{\xi p'}\right)\right]\right\}\right] P_0 \quad (6)$$

2. **Length of the queue ($L_q$)**

$$L_q = \sum_{n=c}^{\infty}(n-c)P_n$$

$$= \sum_{n=c+1}^{\infty}(n-c)\prod_{k=c+1}^{n}\frac{a}{b+(k-c)d} =$$

$$\frac{1}{d(b+d)}\left(\frac{a}{d}\right)^{-\frac{b}{d}}\left(a^2\left(\frac{a}{d}\right)^{b/d} + d^2 e^{a/d}\,\Gamma\left(\frac{b+d}{d}\right) - d^2 e^{a/d}\,\Gamma\left(\frac{b+2d}{d}\right) - \right.$$
$$d^2 e^{a/d}\,\Gamma\left(\frac{b}{d}+1, \frac{a}{d}\right) + d^2 e^{a/d}\,\Gamma\left(\frac{b}{d}+2, \frac{a}{d}\right) + b\,d\,e^{a/d}\,\Gamma\left(\frac{b+d}{d}\right) +$$
$$a\,d\,e^{a/d}\,\Gamma\left(\frac{b+2d}{d}\right) - b\,d\,e^{a/d}\,\Gamma\left(\frac{b+2d}{d}\right) - b\,d\,e^{a/d}\,\Gamma\left(\frac{b}{d}+1, \frac{a}{d}\right) -$$
$$\left. a\,d\,e^{a/d}\,\Gamma\left(\frac{b}{d}+2, \frac{a}{d}\right) + b\,d\,e^{a/d}\,\Gamma\left(\frac{b}{d}+2, \frac{a}{d}\right)\right)$$

Where $a = \lambda(1+\eta), b = c\mu p, d = \xi p'$, $\Gamma(x)$ is gamma function and , $\Gamma(x,y)$ is an incomplete gamma function.

3. **Average time a customer has to wait in the queue ($W_q$):**

$$W_q = \frac{L_q}{\lambda(1+\eta)} \tag{8}$$

4. **Average time a customer has to wait in the system ($W_s$):**

$$W_s = W_q + \frac{1}{c\mu} \tag{9}$$

5. **Length of the system ($L_s$):**

$$L_s = \lambda(1+\eta)W_s \tag{10}$$

## 5 Numerical Illustration

Customers visit a particular firm in accordance to Poisson process with an average rate of 3 customers per unit time. In order to attract customers and to stay ahead of competitors firm wants to introduce lucrative deals and discounts and is expecting 75% increase in the number of customers visiting the firm on the basis of data when firm offered similar deals in the past. There are 3 servers providing service in accordance to exponential distribution with average service rate of 4 customers per unit time. The probability that a dissatisfied customer joins the queue again is given as 0.4 and an impatient customer may leave the queue without taking service with probability 0.2. Firm deployed certain strategies to retain the leaving customers and is expecting that retention probability is 0.3. So as to design strategies for smooth functioning of the system firms needs to measure its performance. Calculate the following:

1. Probability that there is no customer in the system
2. Probability that a customer has to wait
3. Length of the queue
4. Average time a customer has to wait in the queue
5. Average time a customer has to wait in the system
6. Length of the system

**Solution:** Following are the parameters of the above problem:

$\lambda = 3;\ \eta = 0.75; c = 3;\ \mu = 4;\ \xi = 0.2;\ p = 1 - q = 1 - 0.4 = 0.6;\ p' = 1 - q' = 1 - 0.3 = 0.7;$ Using equations (5) to (10), we get:

| | | |
|---|---|---|
| **1** | Probability that there is no customer in the system | **0.0892506** |
| **2** | Probability that a customer has to wait | **0.5019746** |
| **3** | Length of the queue | **1.0159273** |
| **4** | Average time a customer has to wait in the queue | **0.1935100** |
| **5** | Average time a customer has to wait in the system | **0.2768433** |
| **6** | Length of the system | **1.4534273** |

## 6 Conclusions and Future Scope

Since the paper compiles realistic customer behavior, the model can predict the behavior of system. The probabilistic outputs can be used for effective business management. Performance measures derived can prove to be helpful for firms experiencing issues discussed in this paper and can enable firms to develop effective strategies for smooth and efficient functioning of the system.

In future the cost model can be developed and optimization of number of servers or service rate can be performed for efficiency of the system.

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