

THE MATHEMATICAL FRAMEWORK FOR THE PHENOMENON OF OPTICAL CONFINEMENT

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ABSTRACT

Since the introduction of the LASER in 1960, the first laser has been built by Theodore H. Maiman at Hughes Research Laboratories, several attempts have been made to find a mathematical foundation for the unique phenomenon that the speed of light for a LASER beam

equals zero in the transverse directions of propagation. Till now not successfully. It is well-known that Maxwell's famous 4 equation do not have the mathematical Framework to solve this phenomenon of a LASER beam. For this reason, a new electromagnetic theory has been introduced, based on Newton's fundamental principle of equilibrium. Applying Newton's second law of motion at a LASER beam, results in a new mathematical framework which solves this fundamental problem in Physics... The "New Theory" which will be introduced in this article has been based on the fundamental principle of "Perfect Equilibrium within the Universe". A fundamental universal principle in Physics which has already been expressed by Newton's famous 3 equations, published in 1687 in "Philosophiae Naturalis Principia Mathematica. Newton's famous Equations in 3 dimensions will be published in this article in an extension into 4 dimensions. Newton's 4- dimensional law in the 3 spatial dimensions results in an improved version of the classical Maxwell Equations and Newton's law in the 4th dimension (time) results in the quantum mechanical Schrödinger wave equation (at non- relativistic velocities) and the relativistic Dirac equation. We recognize the century of Albert Einstein who triggered in 1905 the large changings in thinking with his famous theory of Special Relativity represented in his publication "On the Electrodynamics of Moving Bodies". Manifesting a "New Theory" and a "New Way of Thinking" with important contributions of Hendrik Lorentz, Henri Poincaré and Hermann Minkowski. It is

recognizable that with the sudden change in thinking in a new period, a new kind of mutual common sense and a general agreement by many scientists of the new theory and the new way of thinking arises. The “New Theory” will be protected by common sense and mutual agreement. This new way of thinking settles down in the scientific society and become immovable. Other options disappear and simply do not exist anymore. Which will make it almost impossible for the following “New Theory” to rise. The “New Theory” which will be introduced in this article has been based on the fundamental principle of “Perfect Equilibrium within the Universe”. A fundamental universal principle in Physics which has already been expressed by Newton’s famous 3 equations, published in 1687 in “Philosophiae Naturalis Principia Mathematica. Newton’s famous Equations in 3 dimensions will be published in this article in an extension into 4 dimensions. Newton’s 4-dimensional law in the 3 spatial dimensions results in an improved version of the classical Maxwell Equations and Newton’s law in the 4th dimension (time) results in the quantum mechanical Schrödinger wave equation (at non-relativistic velocities) and the relativistic Dirac equation.

Index Terms: General Relativity, Classical Electrodynamics, Relativistic Quantum Physics, Electromagnetic-Gravitational Interaction, Dirac Equation, Maxwell Tensor, Energy Momentum Tensor.

INTRODUCTION

The inner structure of a photon is based on a 3-dimensional anisotropic equilibrium within the electromagnetic pulses in which an equilibrium does exist for the Electric and the Magnetic Fields separately generated by the pulses. A photon cannot be considered as a particle. Because particles are 3-dimensional confinements. Photons are anisotropic (in 1st and 2nd dimension a particle and in the 3rd dimension a wave) confinements of electromagnetic pulses, generated during the energy transitions within the atoms. Photons are 2-dimensional confinements of electromagnetic energy and demonstrate the property of inertia (electromagnetic mass) in the 2 directions of confinement. In the 3rd direction, the direction of propagation, photons can only be considered as an electromagnetic wave and for that reason do not demonstrate the property of inertia. All we know about light, and in general about any electromagnetic field configuration, has been based only on two fundamental theories. James Clerk Maxwell introduced in 1865 the “Theory of Electrodynamics” with the publication: “A Dynamical Theory of the Electromagnetic Field”

and Albert Einstein introduced in 1905 the “Theory of Special Relativity” with the publication: “On the Electrodynamics of Moving Bodies” and in 1913 the “Theory of General Relativity” with the publication “Outline of a Generalized Theory of Relativity and of a Theory of Gravitation”. However, both theories are not capable to explain the property of electromagnetic mass and in specific the anisotropy of the phenomenon of electromagnetic mass. To understand what electromagnetic inertia and the corresponding electromagnetic mass is and how the anisotropy of electromagnetic mass can be explained and how it has to be defined, a New Theory about Light has to be developed. A part of this New Theory about Light will be published in this article.

The New Theory about Light has been based on one single fundamental property of our Universe. The unique property that there has always been, is always and will always be a perfect equilibrium within our Universe. Isaac Newton has discovered this fundamental physical law already 300 years ago by his third law in physics. “For every action there is an equal and opposite reaction”. In the New Theory this law of Equilibrium has been extended for any arbitrary Electromagnetic Field Configuration, which requires the fundamental Universal Property: “The total algebraic sum of all force densities will always equal zero at any time at any spatial coordinate in any spatial direction”.

To develop a set of 4 electromagnetic equations, describing all the force densities within any arbitrary electromagnetic field configuration, the Divergence of the 4-Dimensional Stress-Energy Tensor has been taken, resulting in the 4-Dimensional Electromagnetic Vector Equation with the fundamental requirement: “The the total algebraic sum of all force densities equals zero at any time at any spatial coordinate in any spatial direction”

BLACK HOLES

In Maxwell’s time there were no optical LASERS (**L**ight **A**mplification by **S**timulated **E**mission of **R**adiation) and the outcome of his theory was in his time completely in correspondence with what could be measured at that time. The value for the speed of light, calculated from the Maxwell Equations, corresponded almost exactly with the value for the speed of light measured in 1862 by Léon Foucault by a system of rotating mirrors and measured in 1877 by Albert Michelson (300.140 [km/s]).

But nowadays there arises several problems with Maxwell’s theory for the electromagnetic field. Since the existence of the LASERS it became clear that the speed of light is not always

the same in every direction. When a beam of light, generated by a LASER, propagates with the well-known speed of light “ $c = 299.792$ [km/s]” in the z-direction, the speed of light equals zero in the x- direction and the y-direction (in a orthogonal x,y,z frame).

This new phenomenon cannot be explained by Maxwell’s Theory. In Maxwell’s Theory the speed of light has to be exactly the same in every direction. This is clearly not the fact for a LASER beam. And also for the projection of a slide on a screen, it is clearly that the speed of light within the plane of the screen equals zero.

Because the slide we observe does not move. While the projection beam itself moves towards the screen with the speed of light “c”, the beam clearly remains focused and does not move within the plane, perpendicular to the direction of propagation.

Another effect which cannot be explained by Maxwell’s Theory about electromagnetism has been demonstrated within the IBM research group. A new, until 1995 unknown, experiment has been conducted by: O. Gunawan, Y Virgus and K. Fai Tai to demonstrate a subtle hidden feature in electromagnetism - a previously unknown field confinement effect that they named the "camelback effect" (Ref. 1) in a system of two lines of transverse dipoles.

In electromagnetism, the elementary source of electric field and magnetic field can be respectively modeled as a point charge - a hypothetical charge located at a single point in space - and a dipole, a pair of equal and oppositely charged or magnetized poles separated by a distance. Imagine we line up two rows of magnetic dipoles and we try to measure the strength of the magnetic field along the center axis. The magnetic field is certainly stronger at the center and diminishes away from it. However, if the length of the dipole line exceeds certain critical length, a surprising effect occurs: the field gets slightly stronger near the edges and produces a field confinement profile that looks like a camel's back—hence the name of the effect. The IBM team has reported this discovery with detailed experimental and theoretical studies in two recent publications and patents.

This surprising discovery is exciting for a few reasons. First, it represents a new elementary one-dimensional confinement potential in physics, joining the list of well-known potentials such as Coulomb, parabolic, and square well. Second, this effect becomes the key feature that enables this system to serve as a new class of natural magnetic trap (Ref. 2) called parallel dipole line (PDL) trap with many possible exciting applications. This camelback effect and

the related PDL magnetic trap can be realized using special cylindrical magnets whose poles are on the curved side and a graphite rod as the trapped object. This new, until 1995 unknown, effect can only be explained by electromagnetic interaction, described in the New Theory (Ref. 3).

A recent experiment Ref. 22) in 2019 at the Yale University in New Haven C.T. USA published in Nature with the title: *“To catch and and reverse a quantum jump mid-flight”* conflicts a fundamental aspect of the Copenhagen Interpretation related to

“Fundamental Uncertainty” (Probability) represented within the “Standard Model” in Quantum Physics (Ref. 4).

In quantum physics, measurements can fundamentally yield discrete and random results. Emblematic of this feature is Bohr’s 1913 proposal of quantum jumps between two discrete energy levels of an atom. Experimentally, quantum jumps were first observed in an atomic ion driven by a weak deterministic force while under strong continuous energy measurement.

The times at which the discontinuous jump transitions occur are reputed to be fundamentally unpredictable. Despite the non- deterministic character of quantum physics, is it possible to know if a quantum jump is about to occur? Here we answer this question affirmatively: we experimentally demonstrate that the jump from the ground state to an excited state of a superconducting artificial three-level atom can be tracked as it follows a predictable ‘flight’, by monitoring the population of an auxiliary energy level coupled to the ground state. The experimental results demonstrate that the evolution of each completed jump is continuous, coherent and deterministic. The only explanation for this deterministic effect has been described within the new theory which originated from the deterministic electromagnetic field.

There is no other conclusion than the conclusion that the Maxwell Equations are “wrong” or at least “not complete”. The right equation(s) have to describe both possibilities. The possibility that the light moves in every direction with the exactly the same speed of light “c” (like the light being emitted by the sun). And the possibility that the light moves only in one direction and equals zero in the directions perpendicular to the plane of propagation like the propagation of a LASER beam.

A second conclusion can only be that fundamental quantum mechanical relations like the Schrödinger wave equation and the relativistic Dirac equation both originate from a deterministic field like an electromagnetic field which has been demonstrated in the new theory.

To find these new equation(s) we observe that the Maxwell equations are not in unification with Newton's theory of equilibrium of forces. The Maxwell Equations are not in unification with Newton's 3rd law "action = - reaction". Maxwell has not included the force densities with an electromagnetic field at all. To find this new equation, we have to introduce the force densities within an electromagnetic field.

I. Inertia, The Interaction Between Confined Electromagnetic Energy And Gravity

Black Holes are a challenge in modern Physics. In Newton's time it was impossible to speak about a phenomenon like a Black Hole. A place in the Universe in which all the light disappears. Because in Newton's time "Light" was symbolic for "God". And a Black Hole eating and destroying "Light" would be like the Devil eating and destroying "God". Newton immediately would be accused of evil witchcraft and worshipping the Devil. And Newton would have been burnt alive on the stakes because in his time it was still usual to burn witches and other heretics on the medieval stakes.

This fact makes it interesting, because maybe Newton knew about the possibility of Black Holes in the Universe. Because it is well-known that Newton kept many of his secret writings hidden for the scientific world. And the fact that Newton's famous equation of motion turns out to be a theoretical and mathematical foundation for the existence of Black Holes in the universe.

Newton published his famous second law of motion in 1705 in "Philosophiae Naturalis Principia Mathematica" which will turn out to be the foundation for the possible existence of Black Holes in the Universe, when Newton's second law of motion will be applied to a beam of light within a gravitational field.

II. Inertia And The Relative Change In Radiation Pressure

To understand this mathematical approach in physics, it is necessary to understand the concept of "inertia". The property of inertia for "matter" as well the property of inertia for

“light”.



Figure 1: The comparison of a waterjet emitted by a shower head and a laser beam emitted by a laser.

To understand the property of “inertia” for a beam light, we start with an imaginary model of a beam of light, presented by a jet of water leaving a shower head.

Imagine you hold your right hand in the water jet, emitted by a shower head. Then you will feel the radiation pressure of the water jet, comparable with the radiation pressure of a beam of light emitted by a laser.



Figure 2: The shower head emits a waterjet towards to the right and the laser emits a laser beam towards to the right.

When you move your right hand towards to the left (in the opposite direction of the water jet), you will feel that the radiation pressure of the water jet increases. When you move your hand towards to the right (in the direction of the water jet) you will feel that the radiation pressure of the water jet decreases.

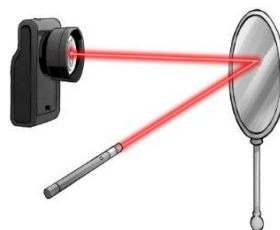


Figure 3: A laser beam has been reflected by the mirror (placed at the right from the laser) and acts with an electromagnetic radiation pressure on the mirror directed

towards to the right.

In a comparable way the radiation pressure of a laser beam emitted by a laser acting on the mirror, will **increase** when we move the mirror towards to the **left** (towards the laser, in the opposite direction of propagation of the laser beam). The radiation pressure on the mirror will **decrease** when we move the mirror towards to the **right** (away from the laser, in the direction of propagation of the laser beam).

Of course, the radiation pressure of the beam of laser light will be much smaller compared to the radiation pressure of the water jet. But with very sensitive equipment the radiation pressure of the beam of laser light can be measured. As an example, the radiation pressure on earth of the light emitted by the sun, equals about $10 \text{ } [\mu\text{N}/ \text{m}^2]$. Which equals a total radiation pressure of the sunlight acting on the surface of the whole earth of about $1.25 \cdot 10^9 \text{ [N]}$.

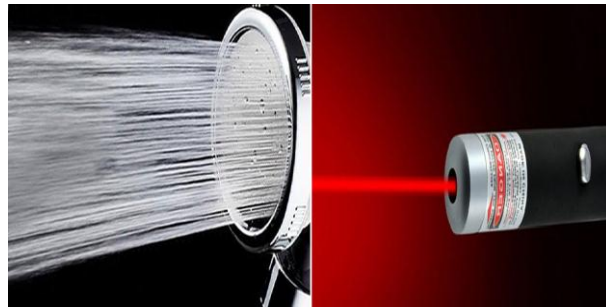


Figure 4: The shower head emits a waterjet towards to the left and the laser emits a laser beam towards to the left.

Now we choose a shower head and a laser emitting towards the left. When you move your left hand towards to the left (in the direction of the water jet), you will feel that the radiation pressure of the water jet decreases. When you move your left hand towards to the right (in the opposite direction of the water jet) you will feel that the radiation pressure of the water jet increases.



Figure 5: A laser beam has been reflected by the mirror (placed at the left from the laser) and acts with an electromagnetic radiation pressure on the mirror directed

towards to the left.

In a comparable way the radiation pressure of a laser beam emitted by a laser acting on the mirror, will decrease when we move the mirror towards to the left (away from the laser, in the direction of propagation of the laser beam). The radiation pressure on the mirror will increase when we move the mirror towards to the right (in the opposite direction of the laser beam).

III. Inertia And The Relative Change Of The Radiation Pressures Of Confined Water Jets

To demonstrate the property of inertia, we place the right hand in the water jet, emitted towards to the right by the first shower head.

The the left hand has been placed in the water jet, emitted towards to the left by the second shower head.

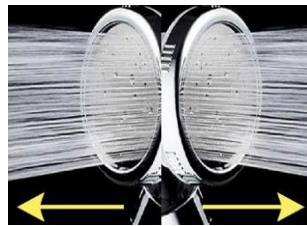


Figure 6: Two shower heads, pointing in opposite directions, emit two waterjets pointing in opposite directions. The left hand has been placed in the waterjet emitted towards to the left and the right hand has been placed in the waterjet emitted towards to the right.

The radiation pressure of the water jet acting on the left hand equals the radiation pressure acting on the right hand but has been directed oppositely. Both radiation pressures neutralize each-other and the resulting force acting on the system of both hands equals zero.

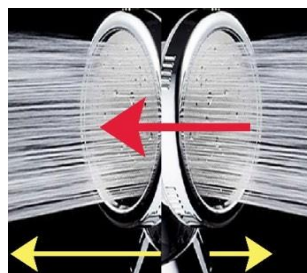


Figure 7: The left hand has been placed in the waterjet emitted towards to the left and the right hand has been placed in the waterjet emitted towards to the right. The

resulting force has been oriented towards to the Left.

To demonstrate the property of inertia of the confined water jets between both hands, we move both hands simultaneously towards to the right. The radiation pressure of the waterjet acting on the left hand will increase. The radiation pressure of the waterjet acting on the right hand will decrease. The total resulting radiation pressures of both waterjets will not neutralize each-other anymore. The total resulting radiation pressure of both waterjets will be directed towards to the left, opposite in the direction of moving. In figure 7 this has been represented by the red arrow pointing towards to the left. We experience the effect of inertia. Both waterjets resist the starting movement of our both hands towards to the right, when we consider both hands together as one mechanical system.

We replace both hands by one (plastic) box, placed over both waterjets. When we move the box towards to the right, we will experience the (extra) inertia of the box, caused by the resulting force of both waterjets directed towards to the left. This is the resulting inertia force, represented by the red arrow in figure 7. Moving the box towards to the right results in an inertia force in the opposite direction represented by the red arrow in figure 7.

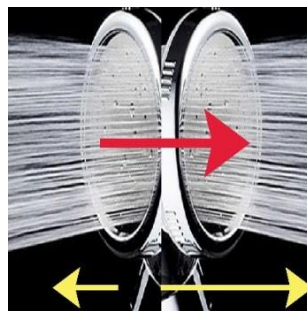


Figure 8: The left hand has been placed in the waterjet emitted towards to the left and the right hand has been placed in the waterjet emitted towards to the right. The resulting force has been oriented towards to the Right.

To demonstrate the property of inertia of the confined water jets between both hands, we move both hands simultaneously towards to the left. The radiation pressure of the waterjet acting on the left hand will decrease. The radiation pressure of the waterjet acting on the right hand will decrease. The total resulting radiation pressures of both waterjets will not neutralize each-other anymore. The total resulting radiation pressure of both waterjets will be directed towards to the left, opposite in the direction of moving. In figure 7 this has been represented by the red arrow pointing towards to the left. We experience the effect of inertia. Both waterjets resist the starting movement of our both hands towards to the left, when we

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IV. Inertia And The Relative Change Of The Radiation Pressures Of Confined Laser Beams

To demonstrate the property of inertia of the electromagnetic radiation (laser beam), confined between both mirrors, we place the right mirror in the laser beam, emitted towards to the right by the first laser. The left mirror has been placed in the laser beam, emitted towards to the left by the second laser.

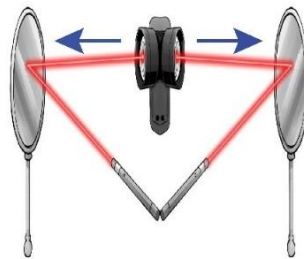


Figure 9: Two lasers, pointing in opposite directions, emit two laser beams pointing in opposite directions. The left mirror has been placed in the laser beam emitted towards to the left and the right mirror been placed in the laser beam towards to the right.

The radiation pressure of the laser beam acting on the left mirror equals the radiation pressure acting on the right mirror but has been directed oppositely. Both radiation pressures neutralize each other, and the resulting force acting on the system of both mirrors equals zero.

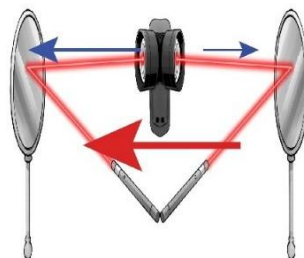


Figure 10: The left mirror has been placed in the laser beam emitted towards to the left and the right mirror been placed in the laser beam towards to the right. The resulting force has been directed towards tot the Left.

To demonstrate the property of inertia of the confined laser beams between between both mirrors, we move both mirrors simultaneously towards to the right. The radiation pressure of the laser beam acting on the left mirror will increase. The radiation pressure of the laser beam acting on the right mirror will decrease. The total resulting radiation pressures of both laser beams will not neutralize each-other anymore. The total resulting radiation pressure of both laser beams will be directed towards to the left, opposite in the direction of moving. In figure 10 this has been represented by the red arrow pointing towards to the left. We experience the effect of inertia. Both laser beams resist the starting movement of our both mirrors, when we consider both mirrors together as one mechanical system.

We replace both mirrors by a box with at the inside 100 % reflecting mirrors. The confined electromagnetic radiation (laser beam) between both mirrors will act on both mirrors with an oppositely directed radiation pressure.

When we start to move the box towards to the right, the resulting total radiation pressure will be oriented towards tot the left. This is the inertia force of the confined electromagnetic radiation. Albert Einstein demonstrated the proportionality between mass (inertia) and energy (intensity of electromagnetic radiation of the confined laser beam) by his famous equation: $E = m c^2$.

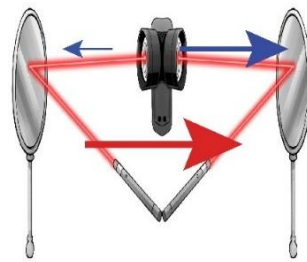


Figure 11: The left mirror has been placed in the laser beam emitted towards to the left and the right mirror been placed in the laser beam towards to the right. The resulting force has been directed towards tot the Right.

To demonstrate the property of inertia of the confined laser beams between between both mirrors, we move both mirrors simultaneously towards to the left. The radiation pressure of the laser beam acting on the left mirror will decrease. The radiation pressure of the laser beam acting on the right mirror will increase. The total resulting radiation pressures of both laser beams will not neutralize each-other anymore. The total resulting radiation pressure of both

laser beams will be directed towards to the right, opposite in the direction of moving. In figure 11 this has been represented by the red arrow pointing towards to the right. We experience the effect of inertia. Both laser beams resist the starting movement of our both mirrors, when we consider both mirrors together as one mechanical system.

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We replace in a “thought experiment” the box with at the inside 100 % reflecting mirrors by a sphere with a 100 % reflecting inside. The hollow sphere contains confined electromagnetic radiation. Proportional to the total confined electromagnetic energy inside the sphere, the sphere will demonstrate the property of “inertia” in any direction according Newton’s second law of motion: $F = m \cdot a$.

Now we replace the hollow sphere with confined electromagnetic radiation by a “Gravitational-Electromagnetic Confinement” of electromagnetic radiation. In a comparable way the electromagnetic confinement will demonstrate the property of “inertia”, represented by Newton’s second law of motion:

$$F = m \cdot a.$$

V. The Anisotropic Effects Of Inertia

A laser beam demonstrates an “anisotropic” effect of inertia. Which means that the effect of inertia is not the same in any direction, like the inertia of confined electromagnetic radiation within a hollow sphere. Only “Confined Electromagnetic Energy” demonstrates the effect of inertia, according Newton’s second law of motion.

In the directions, perpendicular to the direction of propagation of the laser beam, the electromagnetic energy of the Laser beam has been confined. In these directions, perpendicular to the direction of propagation, the laser beam acts according Newton’s second

law of motion $F = m \cdot a$. The mass density of the laser beam has been determined by Einstein's $E = m c^2$. When we divide the left and right terms of Einstein's equation by the Volume, we find a relationship between the energy density "w" and the mass density "ρ". Einstein's equation can be written as: $w = \rho c^2$.

This equation is the fundamental equation for the representation of a "Transversal Black Hole" in which a beam of light follows a circular orbit around a Black Hole due to the Gravity force acting on the mass density "ρ" of the beam of light. Comparable with a satellite moving in a (circular) orbit around the earth because of the gravity force of the earth acting on the mass of the satellite.

In the direction of propagation, the electromagnetic energy has not been confined. In the direction of the propagation of a laser beam does exist a perfect equilibrium between the forward directed radiation pressure and the inertia (mass) of the electromagnetic energy density "ρ". For this reason, it is impossible to accelerate or to decelerate the propagation speed of a laser beam according Newton's second law of motion.

This is comparable with the speed of sound for a sound wave, which will not be influenced by the existence of a gravitational field. A beam of light moving towards a Black Hole or away from a Black Hole will keep the same speed of light. But the intensity of the beam of Light will increase when the beam of light moves towards a Black Hole and the intensity will decrease when a beam of light moves away from a Black Hole. This type of confinement has been called a "Longitudinal Black Hole" in which the confined light always propagates in the direction of the gravitational field of the Black Hole.

VI. The "perfect equilibrium boundary" between "Gravity" and the "radiation pressure"

Light is the phenomenon of electromagnetic radiation, propagating with the speed of light, because of the outside directed radiation pressure. The speed of light has been determined by the "Perfect Equilibrium" between the radiation pressure and the inertia of the electromagnetic mass density.

For spherical waves, the radiation pressure of the electromagnetic energy is equal in any direction and the speed of light is equal in any direction.

For a Laser beam, the radiation pressure has been counterbalanced by electro-magnetic

interaction in the directions perpendicular to the direction of propagation. These are the directions in which the electromagnetic field components, the electric field and the magnetic field, have been oriented. Only in these directions the “electromagnetic interaction” can exist. In the direction perpendicular to these directions, electromagnetic interaction cannot exist. And the radiation pressure will be counterbalanced by the inertia of the electromagnetic energy. This results in the propagation speed of the laser beam perpendicular to the directions of the electric field and the magnetic field with the well-known speed of light.

When a Laser beam enters a gravitational field, the perfect equilibrium between the radiation pressure and the inertia of the electromagnetic energy will be disturbed by the interaction of the gravitational field on the electromagnetic mass density of the Laser beam.

Within a gravitational field the radiation pressure will not change, the electromagnetic interaction will not change, but the total force acting on the electromagnetic mass density of the Laser beam will change.

This results in two effects

- 1) A straight propagation path of a Laser beam will turn into a circular orbit of the Laser beam when the gravitational field has been oriented perpendicular to the direction of propagation.
- 2) The intensity (and frequency) of the Laser beam will decrease or increase when the gravitational field has been oriented in the direction of propagation. But the speed of propagation (speed of light) will not change. Because the radiation pressure represents an alternating force density (with the same frequency as the frequency of the Laser beam) acting on the inertia of the electromagnetic energy. And the gravitational field acts with a stationary force density on the same inertia of the electromagnetic energy.

IX. THE “EVENT HORIZON”

The “event horizon” is the boundary defining the region of space around a black hole from which nothing (not even light) can escape. The “event horizon” has been created by the equilibrium between the gravitational forces and the forces of electromagnetic interaction and the radiation pressure. At the boundary of a Black Hole does exist the perfect equilibrium. There are two types of Black Holes.

The first type represents an equilibrium in which the Black Hole has been represented by a hollow sphere. Only within the extreme thin shell of the hollow sphere does exist a perfect equilibrium. Inside the hollow sphere any kind of light (electromagnetic radiation) has been pressed with an extreme high radiation pressure towards to the “Shell of Perfect Equilibrium” (Event Horizon) because the radiation pressure inside the hollow sphere is always larger than the confining gravitational force. Outside the hollow sphere, any kind of light (electromagnetic radiation) has been attracted towards the “Shell of Perfect Equilibrium” (Event Horizon) because outside the hollow sphere the confining gravitational force is always larger than the radiation pressure. This type has been called “Transversal Black Holes”. Because like a planet moves within an orbit around a star, any beam of light (electromagnetic radiation) propagates within the Shell of Perfect Equilibrium” (Event Horizon) in an orbit around the Black Hole.

The second type represents an Equilibrium everywhere around the black Hole. At a special distance from the center of the Black Hole, the radiation density increases suddenly very sharp. This has been called the Boundary (Event Horizon) around the Black Hole. This type has been called “Longitudinal Black Holes”. Because the light (electromagnetic radiation) moves towards the Black Holes or away from the Black Hole in the same direction as the direction of the Gravitational Field generated by the Black Hole.

X. THE “TRANSVERSAL BLACK HOLE”

We consider a beam of light passing a strong gravitational field, generated by a Black Hole. According the first term in (1) the beam of light will follow a circular orbit around the Black Hole. The required Equilibrium will exist at the radius where the centrifugal electromagnetic inertia forces will be equal and opposite directed to the centripetal oriented gravitational forces on the electromagnetic mass. Figure 12 represents the orbit (colored red) of a LASER beam around a uniform intense gravitational field (Black Hole).

In general, Newtons second law of motion has been presented as:

$$F = m a \quad (1)$$

In which "a" represents the acceleration which equals the difference of the velocity Δv divided by the time interval Δt .

$$a = \frac{\Delta v}{\Delta t} \quad (2)$$

The momentum p of a mechanical mass equals:

$$p = m v \quad (3)$$

Then Newton's second law of motion can be presented as:

$$F_{\text{INERTIA}} = m a = m \frac{\Delta v}{\Delta t} = \frac{\Delta (m v)}{\Delta t} = \frac{\Delta (p)}{\Delta t} \quad (4)$$

(mechanical mass)

Like a mechanical mass expresses the property of inertia, also a beam of light expressed the property of inertia. When the sun shines on the earth, the radiation of the sun presses on the earth with thousands of Newton.

Like a mechanical mass, also a beam of light has momentum. The momentum of a beam of light has been expressed by the Poynting vector S and equals the mechanical momentum vector p multiplied by the square of the speed of light c divided by the Volume.

$$F_{\text{INERTIA}} = m a = m \frac{\Delta v}{\Delta t} = \frac{\Delta (m v)}{\Delta t} = \frac{\Delta (p)}{\Delta t} = \frac{V \Delta (S)}{c^2 \Delta t} \quad (5)$$

(mechanical mass) (beam of light)

The inertia force density “f” equals the inertia force “F” divided by the Volume “V”.

$$f_{\text{INERTIA}} = \left(\frac{m}{V} \right) a = \rho \frac{\Delta v}{\Delta t} = \frac{1}{c^2} \frac{\Delta (S)}{\Delta t} \quad (6)$$

(mechanical mass) (beam of light)

The well-known equation of Einstein equals:

$$W = m c^2$$

$$w = \frac{W}{V} = \frac{m}{V} c^2 = \rho c^2 \quad (7)$$

In which “w” represents the electromagnetic energy density and equals:

$$w = \frac{1}{2} \varepsilon E^2 + \frac{1}{2} \mu H^2 \quad (8)$$

For electromagnetic radiation the electromagnetic impedance Z_0 equals:

$$Z_0 = \frac{E}{H} = \sqrt{\frac{\mu}{\varepsilon}}$$

$$H = E \sqrt{\frac{\varepsilon}{\mu}} \quad (9)$$

$$w = \frac{1}{2} \varepsilon E^2 + \frac{1}{2} \mu H^2 = \frac{1}{2} \varepsilon E^2 + \frac{1}{2} \varepsilon E^2 = \varepsilon E^2$$

Substituting equation (9) in (7) results in:

$$w = \rho c^2 = \varepsilon E^2$$

$$\rho = \frac{\varepsilon}{c^2} E^2 \quad (10)$$

Because the beam of light has been confined in the radial direction, it demonstrates in the radial direction the property of inertia (electromagnetic mass) and interacts with a gravitational field according Newton’s second law of motion. The whole Universe is in a perfect Equilibrium. Also at the “Event Horizon” of a Black Hole does exist a perfect equilibrium between the confining gravitational force of the Black Hole and the radial directed inertia force density of the confined electromagnetic radiation (Laser Beam confined by a Black Hole at the “Event Horizon”).

To determine the “Event Horizon” of the Black Hole (Radius of the circular orbit of the Laser Beam), we have to find the perfect equilibrium between the inertia force densities of the electromagnetic energy densities of the Laser Beam and the confining gravitational force acting on the electromagnetic energy densities of the Laser Beam.

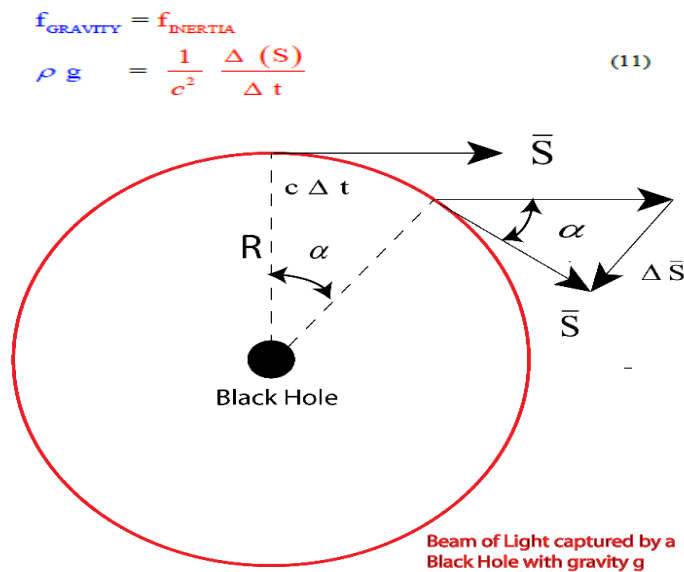


Fig. 12: A LASER beam around a Black Hole captured in a circular orbit around the Black Hole in a Transversal Modus by the Gravitational interaction of the Black Hole with the mass (inertia) of the Laser Beam Light.

From Figure (12) follows the relationship between the changing in the Poynting vector ΔS and the time interval Δt :

$$\text{Tan}(\alpha) = \frac{c \Delta t}{R} = \frac{\Delta S}{S}$$

$$\frac{\Delta S}{\Delta t} = \frac{c S}{R} \quad (12)$$

$$\rho g = \frac{1}{c^2} \frac{\Delta(S)}{\Delta t} = \frac{1}{c^2} \frac{c S}{R} = \frac{1}{c} \frac{S}{R}$$

The Poynting vector \vec{S} represents the total energy transport of the electromagnetic radiation per unit surface per unit time [$\text{J} / \text{m}^2 \text{s}$]. Which can be written as the cross product of the Electric Field intensity \vec{E} and the magnetic Field intensity \vec{H} .

$$\vec{S} = \vec{E} \times \vec{H}$$

$$S = E H \text{Sin}(90^\circ) = E H \quad (13)$$

$$S = E^2 \sqrt{\frac{\epsilon}{\mu}}$$

Substituting equation (10) and (13) in (12) results in an equation for the Event Horizon at radius "R" of a Transversal Black Hole.

$$\begin{aligned}
 \rho g &= \frac{1}{c} \frac{S}{R} \\
 R &= \frac{S}{\rho c g} \\
 R &= \frac{E^2 \sqrt{\frac{\epsilon}{\mu}}}{\rho c g} = \frac{E^2 \sqrt{\frac{\epsilon}{\mu}}}{\frac{\epsilon}{c} E^2 g} \quad (14) \\
 R &= \frac{c^2}{g} \approx \frac{9 \cdot 10^{16}}{g} \quad [\text{m}]
 \end{aligned}$$

Equation (14) represents the perfect equilibrium between the inertia force densities of the electromagnetic mass $\frac{1}{c^2} \frac{\Delta \bar{S}}{\Delta t}$ and the centripetal oriented gravitational force density $\frac{W}{c^2} \bar{g}$ acting on the

Electromagnetic mass. The perfect equilibrium direction [9,10,12,13] where the inertia forces due to the circular orbit of the beam of light are in a perfect balance with the attractive gravitational forces, exists at one defined radius “R” of the beam of light (LASER beam), independent of the intensity of the beam of light and independent of the frequency of the beam of light. Only the acceleration “g” of the gravitational field determines the radius of equilibrium “R”

$$R \approx \frac{9 \cdot 10^{16}}{g} \quad (15)$$

In which “R” is the radius of the beam of light and “g” the acceleration of the gravitational field of the “Black Hole”.

The x-y plane is oriented perpendicular on the z-direction. The speed of light towards the positive z-direction equals the speed of light (the constant "c = 300.000 km/s"). But the speed of light in the x-y plane has to be exactly zero [9,14,15]. Else the diameter of the laser beam would become larger and larger during the propagation along the positive z-direction. This is only possible because the Electromagnetic confining forces B-2, B-3, B-4 and B-5 compensate exactly the outward oriented radiation pressure towards the x-direction and the y-direction.

The Electric Radiation Pressure has been compensated by the Coulomb Force Densities within the Laser Beam

XI. THE “FUNDAMENTAL EQUATION” FOR THE ELECTROMAGNETIC FIELD

In a fundamental way, Newton's second law of motion describes the required electromagnetic equation for the Gravitational-Electromagnetic Interaction for a Longitudinal Black Hole.

Because Maxwell's 4 equations are not part of one "whole uniform understanding of the universe", like the fundamental equation of Newton's second law of motion represents, a "New Equation" has been needed for the Electromagnetic field, based on Newton's second law of motion.

Newton's second law of motion has been based on a profound understanding of the universe which has been based on the fundamental principle of Harmony and Equilibrium.

To describe the interaction between light and gravity it is important to define the fundamental equation for the electromagnetic field based on this fundamental principle of Harmony and Equilibrium formulated by Newton in 1687 and published in his famous work: "Philosophiae Naturalis Principia Mathematica (Mathematical Principles of Natural Philosophy)".

To realize this, Newton's second law of motion will be the ground on which the New Theory will have been built. The fundamental Electromagnetic force density equation has been based integral on Newton's second law of motion and has been divided into 5 separate terms (B-1, B-2, B-3, B-4 and B-5).

The first term B-1 represents the inertia of the mass density of light (Electromagnetic Radiation). The terms B-2 and B-3 represent the electric force densities within the Electromagnetic Radiation (Beam of Light) and the terms B-4 and B-5 represent the magnetic force densities within the Electromagnetic Radiation (Beam of Light).

XII. THE TERM OF INERTIA (TERM B-1)

The right and the left term of Newton's law of motion in equation (16) has been divided by the Volume "V" to find an equation for the force density "f" related to the mass density "ρ".

$$\begin{aligned}
 \mathbf{F} &= m \mathbf{a} \\
 \left(\frac{\mathbf{F}}{V} \right) &= \left(\frac{m}{V} \right) \mathbf{a} \\
 \mathbf{f} &= \rho \mathbf{a}
 \end{aligned} \tag{16}$$

Electromagnetic Inertia Force Density (Term B-1)

$$\begin{aligned}
 F_{INERTIA} &= -m \mathbf{a} = -m \frac{\Delta v}{\Delta t} = -\frac{\Delta(mv)}{\Delta t} = -\frac{\Delta p}{\Delta t} = -\left(\frac{V}{c^2} \right) \frac{\Delta S}{\Delta t} \\
 \frac{F_{INERTIA}}{V} &= -\frac{m}{V} \mathbf{a} = -\frac{m}{V} \frac{\Delta v}{\Delta t} = -\frac{1}{V} \frac{\Delta p}{\Delta t} = -\left(\frac{1}{c^2} \right) \frac{\Delta S}{\Delta t} \\
 \mathbf{f}_{INERTIA} &= -\rho \mathbf{a} = -\left(\frac{1}{c^2} \right) \frac{\Delta S}{\Delta t} \quad [\text{N/m}^3]
 \end{aligned} \tag{17}$$

The Poynting vector $\bar{\mathbf{S}}$ represents the total energy transport of the electromagnetic radiation per unit surface per unit time [J / m² s]. Which can be written as the cross product of the Electric Field intensity $\bar{\mathbf{E}}$ and the magnetic Field intensity $\bar{\mathbf{H}}$.

$$\begin{aligned}
 \mathbf{f}_{INERTIA} &= -\rho \mathbf{a} = -\left(\frac{1}{c^2} \right) \frac{\Delta S}{\Delta t} = -\left(\frac{1}{c^2} \right) \frac{\Delta (\bar{\mathbf{E}} \times \bar{\mathbf{H}})}{\Delta t} \quad [\text{N/m}^3] \\
 \mathbf{f}_{INERTIA} &= -\left(\frac{1}{c^2} \right) \frac{\partial (\bar{\mathbf{E}} \times \bar{\mathbf{H}})}{\partial t} \quad [\text{N/m}^3]
 \end{aligned} \tag{18}$$

XIII. THE ELECTRIC FORCE DENSITY (TERM B-2, COULOM'S LAW)

An example of the Coulomb Force is the Electric Force $F_{Coulomb}$ acting on an electric charge Q placed in an electric field E . The equation for the Coulomb Force equals:

$$\begin{aligned}
 \bar{\mathbf{F}}_{COULOMB} &= \bar{\mathbf{E}} Q \quad [\text{N}] \\
 \text{Dividing both terms by the Volume } V: \\
 \frac{\bar{\mathbf{F}}_{COULOMB}}{V} &= \bar{\mathbf{E}} \frac{Q}{V} \quad [\text{N/m}^3] \\
 \text{Results in the Electric force density:} \\
 \bar{\mathbf{f}}_{COULOMB} &= \bar{\mathbf{E}} \rho_E = \bar{\mathbf{E}} (\nabla \cdot \mathbf{D}) = \epsilon \bar{\mathbf{E}} (\nabla \cdot \mathbf{E}) \quad [\text{N/m}^3]
 \end{aligned} \tag{19}$$

XIV. THE MAGNETIC FORCE DENSITY (TERM B-5, LORENTZ FORCE)

An example of the Lorentz Force is the Magnetic Force $F_{Lorentz}$ acting on an electric charge Q moving with a velocity v within a magnetic field with magnetic field intensity B (magnetic induction). The equation for the Lorentz Force equals:

$$\vec{F}_{\text{LORENTZ}} = Q \vec{v} \times \vec{B} \quad [\text{N}]$$

Dividing both terms by the Volume V:

$$\frac{\vec{F}_{\text{LORENTZ}}}{V} = - \vec{B} \times \frac{Q \vec{v}}{V} = - \vec{B} \times \vec{j} = - \mu \vec{H} \times \vec{j} \quad [\text{N/m}^3] \quad (20)$$

Results in the magnetic force density:

$$\vec{f}_{\text{LORENTZ}} = - \vec{B} \times \frac{Q \vec{v}}{V} = - \vec{B} \times \vec{j} = - \mu \vec{H} \times \vec{j} \quad [\text{N/m}^3]$$

In which q is the electric charge, v the velocity of the electric charge, B the magnetic induction and j the electric current density.

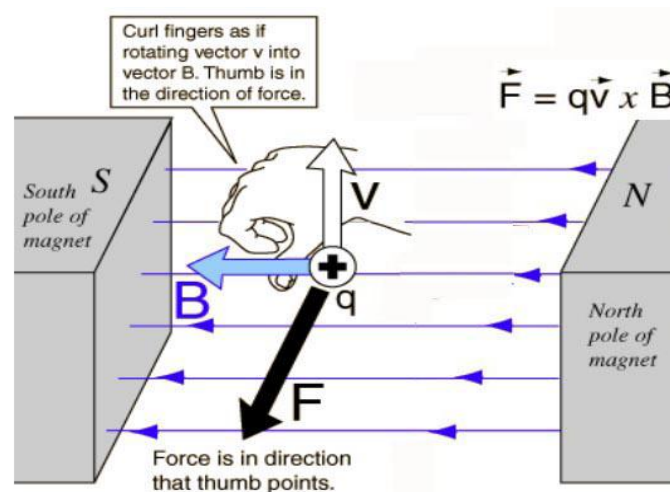


Fig. 13: The Lorentz Force equals the cross product of the Magnetic Induction B and the velocity v of the charge q moving within the magnetic field times the value of the electric charge.

Ampère's Circuital Law represents the electric current density j .

$$\vec{j} = \nabla \times \vec{H} \quad (21)$$

Substituting Ampère's Circuital Law (21) in equation (20) results in equation representing the magnetic force density f_{LORENTZ} :

$$\vec{j} = (\nabla \times \vec{H}) \quad [\text{A/m}^2] \quad (22)$$

$$\vec{f}_{\text{MAGNETIC}} = \vec{f}_{\text{LORENTZ}} = - \mu \vec{H} \times (\vec{j}) = - \mu \vec{H} \times (\nabla \times \vec{H}) \quad [\text{N/m}^3]$$

XV. THE "FUNDAMENTAL EQUATION" FOR THE ELECTROMAGNETIC FIELD

Newton's second law of motion applied within any arbitrary electromagnetic field configuration results in the fundamental equation (23) for any arbitrary electromagnetic field configuration (a beam of light):

$$\begin{aligned}
 & \text{NEWTON: } \mathbf{F}_{\text{TOTAL}} = m \mathbf{a} \text{ represents: } \mathbf{f}_{\text{TOTAL}} = \rho \mathbf{a} \\
 & -\rho \mathbf{a} + \mathbf{f}_{\text{TOTAL}} = 0 \\
 & -\rho \mathbf{a} + \mathbf{f}_{\text{ELEKTRISCH}} + \mathbf{f}_{\text{MAGNETISCH}} = 0 \quad (23) \\
 & -\rho \mathbf{a} + \mathbf{F}_{\text{COULOMB}} + \mathbf{F}_{\text{LORENTZ}} + \mathbf{F}_{\text{COULOMB}} + \mathbf{F}_{\text{LORENTZ}} = 0 \\
 & -\frac{1}{c^2} \frac{\partial (\mathbf{E} \times \mathbf{H})}{\partial t} + \epsilon_0 \mathbf{E} (\nabla \cdot \mathbf{E}) - \epsilon_0 \mathbf{E} \times (\nabla \times \mathbf{E}) + \mu_0 \mathbf{H} (\nabla \cdot \mathbf{H}) - \mu_0 \mathbf{H} \times (\nabla \times \mathbf{H}) = 0 \\
 & \quad \text{B-1} \quad \quad \text{B-2} \quad \quad \text{B-3} \quad \quad \text{B-4} \quad \quad \text{B-5}
 \end{aligned}$$

Term B-4 is the magnetic representation of the (electric) Coulomb's law B-2 and Term B-3 is the electric representation of the (magnetic) Lorentz Force.

XVI. THE "FUNDAMENTAL EQUATION" FOR THE BLACK HOLE, DESCRIBING "GRAVITATIONAL-ELECTROMAGNETIC INTERACTION"

The fundamental equation for a Black Hole describes the interaction between light and gravity. To define the Fundamental Equation for the Black Hole, describing the interaction of Gravity on an electromagnetic field, an extra term (B-6) has been introduced in equation (23). The term B-6 represents the force density of the gravitational field acting on the electromagnetic mass density.

$$F_{\text{GRAVITY}} = m \bar{g} \text{ [N]}$$

Dividing both terms by the Volume V:

$$\frac{F_{\text{GRAVITY}}}{V} = \frac{m}{V} \bar{g} \text{ [N/ m}^3\text{]} \quad (24)$$

Results in the force density:

$$\mathbf{f}_{\text{GRAVITY}} = \rho \bar{g} \text{ [N/ m}^3\text{]}$$

The specific mass "ρ" of a beam of light follows from Einstein's equation:

$$W = m c^2$$

Dividing both terms by the Volume V results in:

$$\frac{W}{V} = \frac{m}{V} c^2$$

which represents the energy density w and the specific mass ρ of the electromagnetic radiation: (25)

$$w = \rho c^2$$

which results for an expression of the specific mass ρ:

$$\rho = \frac{1}{c^2} w = \epsilon \mu w$$

The energy density "w" follows from the electric and the magnetic field intensities:

$$w = \frac{1}{2} \epsilon E^2 + \frac{1}{2} \mu H^2 \quad (26)$$

$$w = \frac{1}{2} (\epsilon E^2 + \mu H^2) = \frac{1}{2} (\epsilon (\bar{E} \cdot \bar{E}) + \mu (\bar{H} \cdot \bar{H}))$$

Substituting equation (26) in equation (17) results in the gravitational force density fGRAVITY acting on an arbitrary electromagnetic field configuration (a beam of light) with mass density ρ .

$$f_{\text{GRAVITY}} = \rho \bar{g}$$

$$f_{\text{GRAVITY}} = \rho \bar{g} = \epsilon \mu w \bar{g} = \frac{1}{2} (\epsilon^2 \mu (\bar{E} \cdot \bar{E}) + \epsilon \mu^2 (\bar{H} \cdot \bar{H})) \bar{g} \quad (27)$$

Substituting equation (27) in equation (23) results in the fundamental equation describing the Electromagnetic-Gravitational interaction for any arbitrary electromagnetic field configuration (a beam of light):

$$\begin{aligned} & \text{NEWTON: } \bar{F}_{\text{TOTAL}} = m \bar{a} \text{ [N]} \\ & \text{NEWTON: Expressed in force densities: } \bar{f}_{\text{TOTAL}} = \rho \bar{a} \text{ [N/m}^3\text{]} \\ & -\rho \bar{a} + \bar{f}_{\text{TOTAL}} = \bar{0} \\ & -\rho \bar{a} + \bar{f}_{\text{ELEKTRECH}} + \bar{f}_{\text{MAGNETISCH}} + \bar{f}_{\text{GRAVITY}} = \bar{0} \\ & \bar{f}_{\text{INERTIA}} + \bar{f}_{\text{COULOMB}} + \bar{f}_{\text{LORENTZ}} + \bar{f}_{\text{COULOMB}} + \bar{f}_{\text{LORENTZ}} + \bar{f}_{\text{GRAVITY}} = \bar{0} \quad (28) \\ & \underbrace{-\frac{1}{c^2} \frac{\partial (\bar{E} \times \bar{H})}{\partial t}}_{\text{B-1}} + \underbrace{\epsilon_1 \bar{E} (\nabla \cdot \bar{E}) - \epsilon_1 \bar{E} \times (\nabla \times \bar{E})}_{\text{B-2}} + \underbrace{\mu_1 \bar{H} (\nabla \cdot \bar{H}) - \mu_1 \bar{H} \times (\nabla \times \bar{H})}_{\text{B-3}} + \underbrace{\frac{1}{2} (\epsilon^2 \mu (\bar{E} \cdot \bar{E}) + \epsilon \mu^2 (\bar{H} \cdot \bar{H})) \bar{g}}_{\text{B-6}} = \bar{0} \end{aligned}$$

Term B-1 represents the inertia term of the electromagnetic radiation. Term B-4 is the magnetic representation of the (electric) Coulomb's Force B-2 and Term B-3 is the electric representation of the (magnetic) Lorentz Force B-5. Term B-6 represents the Electromagnetic-Gravitational interaction of a gravitational field with field acceleration acting on an arbitrary electromagnetic field configuration (a beam of light) with specific mass ρ .

XVII. LONGITUDINAL BLACK HOLES

When a beam of light is approaching a strong gravitational field in the direction of the gravitational field, generated by a Black Hole, the confinement has been called a

Longitudinal Black Hole. The direction of propagation of the beam of light is in the same direction (or in the opposite direction) of the gravitational field. According the first term in (1), the beam of light will be accelerated or decelerated. However, the speed of light is a universal constant and for that reason the speed of light cannot increase or decrease. Instead the intensity of the electromagnetic radiation will increase when the beam of light approaches (propagates in the opposite direction as the direction of the gravitational field) the Black Hole. And the intensity of the electromagnetic radiation will decrease when the beam of light leaves (propagates in the same direction as the direction of the gravitational field) the Black Hole.

The Gravitational-Electromagnetic Confinement for the elementary structure for the “Longitudinal Black Hole” has been presented in equation (29).

$$\begin{array}{c}
 \text{3-Dimensional Space Domain} \\
 \text{B-1} \qquad \qquad \text{B-2} \qquad \qquad \text{B-3} \\
 \left(\begin{array}{c} x_3 \\ x_2 \\ x_1 \end{array} \right) + \underbrace{-\frac{1}{c^2} \frac{\partial (\vec{E} \times \vec{H})}{\partial t}}_{\text{B-4}} + \underbrace{\epsilon_0 \vec{E} (\nabla \cdot \vec{E})}_{\text{B-5}} - \underbrace{\epsilon_0 \vec{E} \times (\nabla \times \vec{E})}_{\text{B-6}} + \\
 + \underbrace{\mu_0 \vec{H} (\nabla \cdot \vec{H})}_{\text{B-4}} - \underbrace{\mu_0 \vec{H} \times (\nabla \times \vec{H})}_{\text{B-5}} + \frac{1}{2} (\epsilon^2 \mu (\vec{E} \cdot \vec{E}) + \epsilon \mu^2 (\vec{H} \cdot \vec{H})) \vec{g} = \vec{0} \quad \text{In}
 \end{array} \quad (29)$$

Which represents the gravitational acceleration acting on the the electromagnetic mass density of the confined electromagnetic radiation. g

A possible solution for equation (29) describing an Electromagnetic-Gravitational confinement within a radial gravitational field with acceleration has been represented in (30).

$$\begin{pmatrix} e_r \\ e_\theta \\ e_\varphi \end{pmatrix} = \begin{pmatrix} 0 \\ f(r) \sin(\omega t) \\ -f(r) \cos(\omega t) \end{pmatrix} \quad \begin{pmatrix} m_r \\ m_\theta \\ m_\varphi \end{pmatrix} = \begin{pmatrix} 0 \\ f(r) \cos(\omega t) \\ f(r) \sin(\omega t) \end{pmatrix} \quad \vec{g} = \begin{pmatrix} \frac{G_1}{4 \pi r^2} \\ 0 \\ 0 \end{pmatrix} \quad (30)$$

$$w_{em} = \left(\frac{\mu_0}{2} (\vec{m} \cdot \vec{m}) + \frac{\epsilon_0}{2} (\vec{e} \cdot \vec{e}) \right) = \epsilon_0 f(r)^2$$

In which the radial function $f(r)$ equals:

$$f[r] = K e^{-\frac{-\frac{G1 \varepsilon_0 \mu_0}{r} + 8 \pi \log[r]}{8 \pi}} \quad (31)$$

The solution has been calculated according Newton's Shell Theorem.

XVIII. LONGITUDINAL BLACK HOLE WITH AN ELECTROMAGNETIC MASS OF 10^{-4} [KG] AND A RADIUS EQUALS 2×10^{-35} [M]

Figure 14 and Fig. 15 represent the electromagnetic field density (along the vertical axis) as a function of the distance (along the horizontal axis) of the center of the Longitudinal Black Hole with the size of Planck's length. The chosen values equal:

$$f[r] = K e^{-\frac{-\frac{G1 \text{ emm} \varepsilon_0 \mu_0}{r} + 8 \pi \log[r]}{8 \pi}} \quad (32)$$

$$G1 = 6.6740810^{-11}$$

$$\text{emm} = 10^{-4}$$

$$\varepsilon_0 = 8.8510^{-12}$$

$$\mu_0 = 1.256637061435917210^{-6}$$

In which "emm" represents the electromagnetic mass of the confinement located at the center according Newton's Shell Theorem.

For an electromagnetic mass $\text{emm} = 10^{-4}$ [kg] of the Longitudinal Black Hole, the radius of the confinement equals approximately 2×10^{-35} [m] and the first harmonic frequency equals 1.5×10^{27} [Hz].

The Plot graph of the Electric Field Intensity $f(r)$ of the confinement has been presented as a function of the radius in figure (14) and figure (15):

$$\text{Plot} \left[e^{-\frac{-\frac{G1 \varepsilon_0 \mu_0}{r} + 8 \pi \log[r]}{8 \pi}}, \{r, 10^{-36}, 10^{-25}\} \right]$$

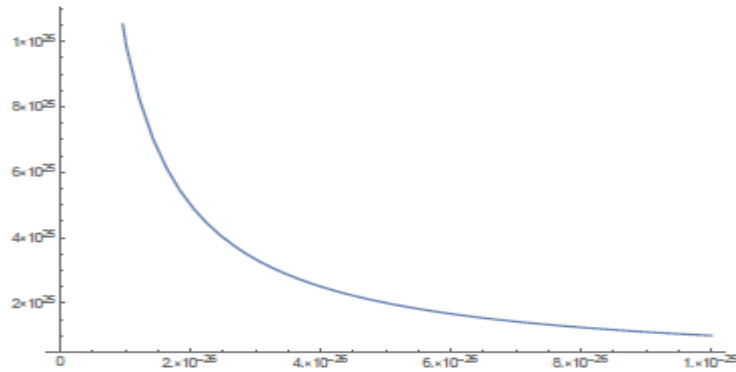


Figure 14: PlotGraph of the Electric Field Intensity $f(r)$ for the region $10^{-36} < r < 10^{-25}$ in which the gravitational field acceleration has been chosen accordingly an electromagnetic mass of 10^{-4} [kg] located at the center of the confinement, according Newton's Shell Theorem.

$$\text{Plot} \left[e^{-\frac{G I \epsilon_0 \mu_0}{r} + 8 \pi \log[r]} , \{r, 10^{-36}, 10^{-35}\} \right]$$

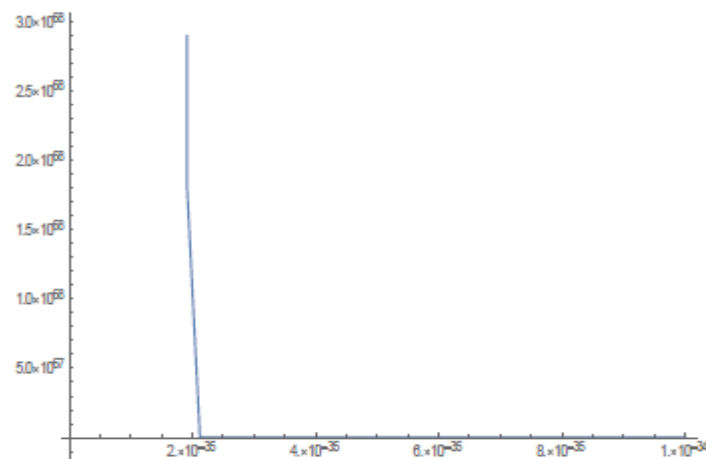


Figure 15: PlotGraph of the Electric Field Intensity $f(r)$ for the region $10^{-36} < r < 10^{-35}$ in which the gravitational field acceleration has been chosen accordingly an electromagnetic mass of 10^{-4} [kg] located at the center of the confinement, according Newton's Shell Theorem.

It follows from Figure 15 that the radius of the stable gravitational electromagnetic confinement equals approximately 2×10^{-35} [m], which is the size of the Planck length.

According the theory of superstrings, the fundamental constituents of reality are strings of the Planck length (about $1.62 \cdot 10^{-35}$ [m]) that vibrate at resonant frequencies.

XIX. LONGITUDINAL BLACK HOLE WITH AN ELECTROMAGNETIC MASS OF 1040 [KG] AND A RADIUS OF $1.5 \cdot 10^9$ [M] A FREQUENCY OF 0.2 [HZ]

Conventional Black Hole with an Electro Magnetic Mass: $emm = 1040$ [kg], the solution (30) and (31) for the Gravitational Electromagnetic Equilibrium Equation (29) results in a Gravitational Electromagnetic Confinement radius $r = 1.5 \cdot 10^9$ [m] (Figure 16 and Figure 17).

$$f[r] = K e^{-\frac{G1 \text{ emm } \epsilon_0 \mu_0}{r} + 8 \pi \log[r]}$$

$$G1 = 6.6740810^{-11}$$

$$\text{emm} = 10^{40}$$

$$\epsilon_0 = 8.8510^{-12}$$

$$\mu_0 = 1.256637061435917210^{-6}$$
(33)

In which “emm” equals the electromagnetic mass of the Single Harmonic Black Hole located at the center according Newton’s Shell Theorem. For an electromagnetic mass of the Single Harmonic Black Hole (SHBH), the value for the electromagnetic mass (emm) equals: $emm = 1040$ [kg], the radius of the confinement equals approximately $1.5 \cdot 10^9$ [m] and the first harmonic frequency equals 0.2 [Hz].

The Plot graph of the Electric Field Intensity $f(r)$ of the SHBH has been presented as a function of the radius in figure (15) and figure (16):

$$\text{Plot} \left[e^{-\frac{G1 \epsilon_0 \mu_0}{r} + 8 \pi \log[r]}, \{r, 10^5, 10^{12}\} \right]$$

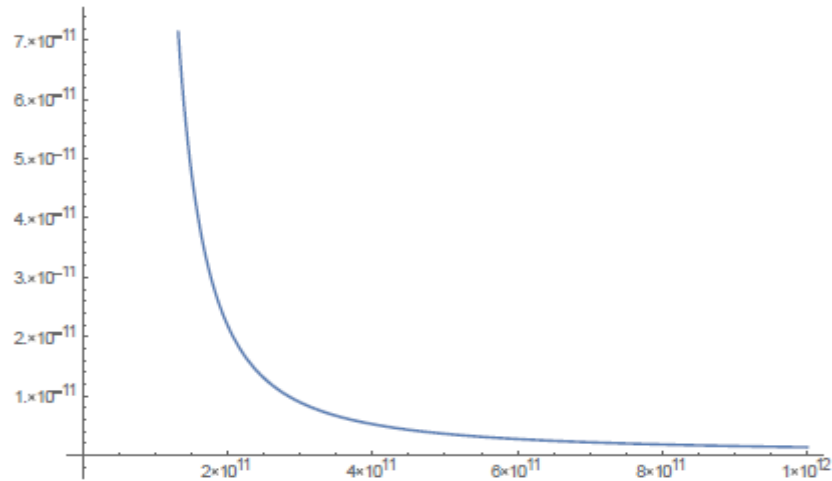


Fig. 16: PlotGraph of the Electric Field Intensity $f(r)$ (vertical axis) for the region $10^5 < r < 10^{12}$ (horizontal axis) in which the gravitational field acceleration has been chosen accordingly an electromagnetic mass of 1040 [kg] located at the center of the confinement, according Newton’s Shell Theorem.

$$\begin{aligned}
 & \text{Energy-Time Domain} \\
 & \text{Inner Energy} \\
 & \text{B-7} \\
 (f_4) \quad & \nabla \cdot \vec{S} + \frac{\partial \mathcal{W}}{\partial t} = 0 \\
 & \text{3-Dimensional Space Domain} \\
 & \text{B-1} \quad \text{B-2} \quad \text{B-3} \\
 & -\frac{1}{c^2} \frac{\partial (\vec{E} \times \vec{H})}{\partial t} + \epsilon_0 \vec{E} (\nabla \cdot \vec{E}) - \epsilon_0 \vec{E} \times (\nabla \times \vec{E}) + \\
 & \text{B-4} \quad \text{B-5} \quad \text{B-6} \\
 & \begin{pmatrix} f_3 \\ f_2 \\ f_1 \end{pmatrix} + \mu_0 \vec{H} (\nabla \cdot \vec{H}) - \mu_0 \vec{H} \times (\nabla \times \vec{H}) + \frac{1}{2} (\epsilon^2 \mu (\vec{E} \cdot \vec{E}) + \epsilon \mu^2 (\vec{H} \cdot \vec{H})) \vec{e} - \vec{0}
 \end{aligned} \tag{34}$$

$$\text{Plot} \left[e^{-\frac{G1 \epsilon_0 \mu_0 + 8 \pi \log[r]}{8 \pi}}, \{r, 10^9, 4 \times 10^9\} \right]$$

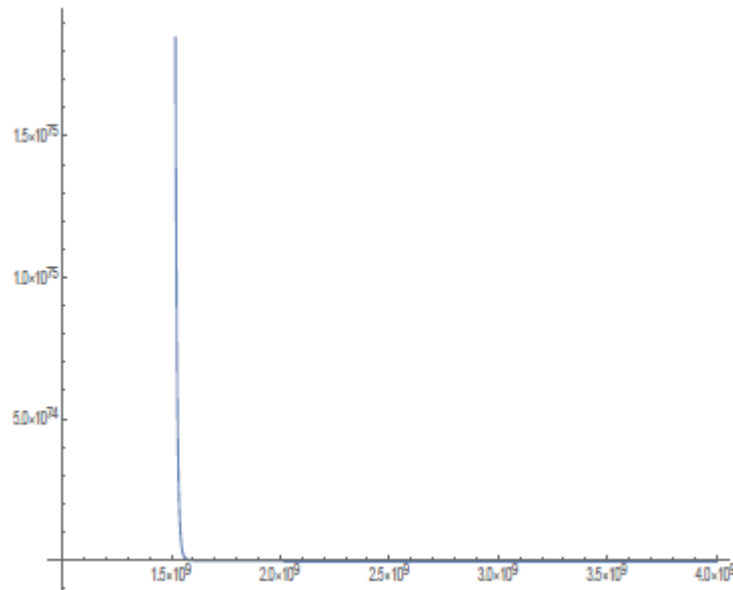


Fig. 17: PlotGraph of the Electric Field Intensity $f(r)$ (vertical axis) for the region $10^9 < r < 4 \cdot 10^9$ (horizontal axis) in which the gravitational field acceleration has been chosen accordingly an electromagnetic mass of 1040 [kg] located at the center of the confinement, according Newton’s Shell Theorem. And a corresponding Single Harmonic frequency of 0.2 [Hz].

It follows from Figure 17 that the radius of the stable gravitational electromagnetic confinement of the SHBH (Single Harmonic Black Hole) equals approximately $1.5 \cdot 10^9$ [m].

XX. QUANTUM GRAVITY DESCRIBED BY NEWTON’S SECOND LAW OF MOTION IN THE 4TH DIMENSION (TIME DOMAIN)

Newton’s second law of motion has been described in 3 spatial dimensions, resulting in the fundamental equation for the electromagnetic field.

$$\begin{matrix}
 & & \text{3-Dimensional Space Domain} & & \\
 & \text{B-1} & & \text{B-2} & \text{B-3} & \\
 \begin{pmatrix} x_3 \\ x_2 \\ x_1 \end{pmatrix} & - \frac{1}{c^2} \frac{\partial (\vec{E} \times \vec{H})}{\partial t} & + \epsilon_0 \vec{E} (\nabla \cdot \vec{E}) & - \epsilon_0 \vec{E} \times (\nabla \times \vec{E}) & + & \\
 & \text{B-4} & & \text{B-5} & \text{B-6} & \\
 & + \mu_0 \vec{H} (\nabla \cdot \vec{H}) & - \mu_0 \vec{H} \times (\nabla \times \vec{H}) & + \frac{1}{2} (e^2 \mu (\vec{E} \cdot \vec{E}) + \epsilon \mu^2 (\vec{H} \cdot \vec{H})) & \vec{g} = \vec{0} & (35)
 \end{matrix}$$

It is possible to calculate the 4th dimension of Newton’s second law of motion by 4-dimensional vector calculus. The 4-dimensional Electromagnetic Vector Potential has been defined by:

$$\vec{\varphi}^4 = \begin{pmatrix} \varphi_4 \\ \varphi_3 \\ \varphi_2 \\ \varphi_1 \end{pmatrix} \xrightarrow{\text{CartesianCoordinateSystem}} \begin{pmatrix} \varphi_t \\ \varphi_z \\ \varphi_y \\ \varphi_x \end{pmatrix} \quad (36)$$

In which the term φ^a represents the 4-dimensional electromagnetic vector potential in the “a” direction while the indice “a” varies from 1 to 4. In a cartesian coordinate system the indices are chosen varying from the x,y,z and t direction. In which the indice “t” represents the time direction which has been considered to be the 4th dimension. The 4-dimensional Electromagnetic “Maxwell Tensor” has been defined by:

$$F_{ab} = \partial_b \varphi_a - \partial_a \varphi_b \quad (37)$$

Where the indices “a” and “b” vary from 1 to 4. The 4-dimensional Electromagnetic “Energy Momentum Tensor” has been defined by:

$$T^{ab} = \frac{1}{\mu_0} \left[F_{ac} F^{cb} + \frac{1}{4} \delta_{ab} F_{cd} F^{cd} \right] \quad (38)$$

The 4-dimensional divergence of the 4-dimensional Energy Momentum Tensor equals the 4-dimensional Force Density 4-vector: f^a .

$$f^a = \partial_b T^{ab} \quad (39)$$

Substituting the electromagnetic values for the electric field intensity “E” and the magnetic field intensity “H” in (39) results in the 4-dimensional representation of Newton’s second law of motion:

$$\begin{aligned}
 & \text{Energy-Time Domain} \\
 & \text{B-7} \\
 (f_4) \quad & \nabla \cdot (\bar{E} \times \bar{H}) + \frac{1}{2} \frac{\partial (\epsilon_0 (\bar{E} \cdot \bar{E}) + \mu_0 (\bar{H} \cdot \bar{H}))}{\partial t} = 0 \\
 & \text{3-Dimensional Space Domain} \\
 & \text{B-1} \quad \text{B-2} \quad \text{B-3} \quad (40) \\
 \begin{pmatrix} f_3 \\ f_2 \\ f_1 \end{pmatrix} & \begin{aligned} & -\frac{1}{c^2} \frac{\partial (\bar{E} \times \bar{H})}{\partial t} + \epsilon_0 \bar{E} (\nabla \cdot \bar{E}) - \epsilon_0 \bar{E} \times (\nabla \times \bar{E}) + \\ & \text{B-4} \quad \text{B-5} \quad \text{B-6} \\ & + \mu_0 \bar{H} (\nabla \cdot \bar{H}) - \mu_0 \bar{H} \times (\nabla \times \bar{H}) + \frac{1}{2} (\epsilon^2 \mu (\bar{E} \cdot \bar{E}) + \epsilon \mu^2 (\bar{H} \cdot \bar{H})) \bar{g} = \bar{0} \end{aligned}
 \end{aligned}$$

Dimensions and f_4 represent the force density in the time dimension (4th dimension).

The 4th term in equation (40) can be written in the terms of the Poynting vector “S” and the energy density “w” representing the electromagnetic law for the conservation of energy.

The relativistic fundamental equation describing Quantum Physics is the Dirac Equation, which equals the 4th term of Newton’s second law of motion.

The 4th term of Newton’s force density equation (41) can be written as the law for the conservation of electromagnetic energy or can be presented as the relativistic quantum mechanical Dirac Equation.

$$\begin{aligned}
 & \text{Energy-Time Domain (x-4)} \\
 & \nabla \cdot (\bar{E} \times \bar{H}) + \frac{1}{2} \frac{\partial (\epsilon_0 (\bar{E} \cdot \bar{E}) + \mu_0 (\bar{H} \cdot \bar{H}))}{\partial t} \\
 \bar{\phi} \cdot \bar{\phi}^* &= \frac{1}{2\mu} \left(\bar{B} + i \frac{\bar{E}}{c} \right) \cdot \left(\bar{B} - i \frac{\bar{E}}{c} \right) = \frac{1}{2} \mu H^2 + \frac{1}{2} \epsilon E^2 = w \\
 \bar{\phi} \times \bar{\phi}^* &= \frac{1}{2\mu} \left(\bar{B} + i \frac{\bar{E}}{c} \right) \times \left(\bar{B} - i \frac{\bar{E}}{c} \right) = i \sqrt{\epsilon \mu} \bar{E} \times \bar{H} = i \sqrt{\epsilon \mu} \bar{S} \\
 -\frac{i}{\sqrt{\epsilon_0 \mu_0}} \nabla \cdot (\bar{\phi} \times \bar{\phi}^*) &= -\frac{\partial \bar{\phi} \cdot \bar{\phi}^*}{\partial t} \\
 \bar{\alpha} = \begin{bmatrix} 0 & \sigma \\ \sigma & 0 \end{bmatrix} \quad \text{and} \quad \bar{\beta} = \begin{bmatrix} \delta_x & 0 \\ 0 & -\delta_x \end{bmatrix} \\
 \left(\frac{imc}{h} \bar{\beta} + \bar{\alpha} \cdot \nabla \right) \psi &= -\frac{1}{c} \frac{\partial \psi}{\partial t} - \frac{g}{c^2} \psi f
 \end{aligned} \quad (41)$$

The 4th term in the electromagnetic representation (41) represents the electromagnetic law for the conservation of energy.

Energy-Time Domain (x-4)

$$\nabla \cdot (\bar{\mathbf{E}} \times \bar{\mathbf{H}}) + \frac{1}{2} \frac{\partial (\epsilon_0 (\bar{\mathbf{E}} \cdot \bar{\mathbf{E}}) + \mu_0 (\bar{\mathbf{H}} \cdot \bar{\mathbf{H}}))}{\partial t} \quad (42)$$

In the quantum physical presentation equation (41) can be written as:

Energy-Time Domain (x-4)

$$\left(\frac{i m c}{h} \bar{\beta} + \bar{\alpha} \cdot \nabla \right) \psi = - \frac{1}{c} \frac{\partial \psi}{\partial t} - \frac{g}{c^2} \psi \quad (43)$$

Which represents the relativistic quantum mechanical Dirac Equation where represents the quantum mechanical probability wave function. The mathematical evidence for the equivalent for (41) and (42) has been published in 1995 in the article: "A Continuous Model of Matter based on AEONs". Equation (1) page 201 to Equation (102) page 213. (Doi: 10.31219/osf.io/ra7ng) ψ .

Both equations (42) and (43) are identical but written in a different way.

The Electromagnetic Law for the conservation of Energy (42) and the Relativistic Dirac Equation (43) change within a gravitational field.

As an example, we consider the experiment of a ball being thrown up within a gravitational field. The kinetic energy of the ball at the beginning will be transformed into potential energy at the end until the ball stops moving and falls back to earth.

Something comparable will happen with a beam of light. We consider the experiment of a flashlight, shing up straight from earth in the same direction of the earth as the gravitational field. During the propagation of the light, the kinetic energy of the light will be transformed into potential energy. The kinetic energy of the light is contained by the electromagnetic field. The speed of the light will not change during the process of propagation, but the intensity of the electric field as well the intensity of the magnetic field will decrease with the same amount as the potential energy of the beam of light will increase. The potential energy of the beam of light has been expressed by the force density of the gravitational field on the mass density of the beam of light multiplied by the distance. Which has been expressed in equation (44):

$$\begin{aligned}
 & \text{Energy-Time Domain (x-4)} \\
 & \nabla \cdot (\bar{E} \times \bar{H}) + \frac{1}{2} \frac{\partial (\epsilon_0 (\bar{E} \cdot \bar{E}) + \mu_0 (\bar{H} \cdot \bar{H}))}{\partial t} + \frac{\sqrt{\epsilon \mu}}{2} (\epsilon (\bar{E} \cdot \bar{E}) + \mu (\bar{H} \cdot \bar{H})) \bar{g} \\
 & \bar{\phi} \cdot \bar{\phi}^* = \frac{1}{2\mu} \left(\bar{B} + i \frac{\bar{E}}{c} \right) \cdot \left(\bar{B} - i \frac{\bar{E}}{c} \right) = \frac{1}{2} \mu H^2 + \frac{1}{2} \epsilon E^2 = w \\
 & \bar{\phi} \times \bar{\phi}^* = \frac{1}{2\mu} \left(\bar{B} + i \frac{\bar{E}}{c} \right) \times \left(\bar{B} - i \frac{\bar{E}}{c} \right) = i \sqrt{\epsilon \mu} \bar{E} \times \bar{H} = i \sqrt{\epsilon \mu} \bar{S} \\
 & -\frac{i}{\sqrt{\epsilon_0 \mu_0}} \nabla \cdot (\bar{\phi} \times \bar{\phi}^*) = -\frac{\partial \bar{\phi} \cdot \bar{\phi}^*}{\partial t} \\
 & \bar{\alpha} = \begin{bmatrix} 0 & \sigma \\ \sigma & 0 \end{bmatrix} \quad \text{and} \quad \bar{\beta} = \begin{bmatrix} \delta_x & 0 \\ 0 & -\delta_x \end{bmatrix} \\
 & \left(\frac{i m c}{h} \bar{\beta} + \bar{\alpha} \cdot \nabla \right) \psi = -\frac{1}{c} \frac{\partial \psi}{\partial t} - \frac{g}{c^2} \psi
 \end{aligned} \tag{44}$$

Equation (44) represents as well “The Electromagnetic Law for the conservation of Energy within a Gravitational Field with acceleration g” and the Relativistic Dirac Equation within a Gravitational Field with acceleration g”.

The complete fundamental equation describing Gravitational-Electromagnetic Interaction within a gravitational field with acceleration “g” is the 4-dimensional representation of Newton’s second law of motion represented in equation (45):

$$\begin{aligned}
 & \text{Energy-Time Domain} \\
 & \text{Inner Energy} \quad \text{Potential Energy} \\
 & \text{B-7} \quad \text{B-8} \\
 & \left(x_4 \right) \quad \nabla \cdot \bar{S} + \frac{\partial w}{\partial t} + m \bar{g} \frac{\Delta x}{\Delta t} = 0 \\
 & \text{3-Dimensional Space Domain} \\
 & \text{B-1} \quad \text{B-2} \quad \text{B-3} \\
 & \left(\begin{matrix} x_3 \\ x_2 \\ x_1 \end{matrix} \right) \quad -\frac{1}{c^2} \frac{\partial (\bar{E} \times \bar{H})}{\partial t} + \epsilon_0 \bar{E} (\nabla \cdot \bar{E}) - \epsilon_0 \bar{E} \times (\nabla \times \bar{E}) + \\
 & \quad \text{B-4} \quad \text{B-5} \quad \text{B-6} \\
 & + \mu_0 \bar{H} (\nabla \cdot \bar{H}) - \mu_0 \bar{H} \times (\nabla \times \bar{H}) + \frac{1}{2} (\epsilon^2 \mu (\bar{E} \cdot \bar{E}) + \epsilon \mu^2 (\bar{H} \cdot \bar{H})) \bar{g} = \bar{0}
 \end{aligned} \tag{45}$$

The fundamental equation, describing “Quantum Gravity” has been represented by the Relativistic Quantum Mechanical Dirac equation with a gravitational field with acceleration g, presented in equation (46):

$$\left(\frac{i m c}{h} \bar{\beta} + \bar{\alpha} \cdot \nabla \right) \psi = -\frac{1}{c} \frac{\partial \psi}{\partial t} - \frac{g}{c^2} \psi \tag{46}$$

XXI. DATA AVAILABILITY

All the Data and all the Calculations to provide evidence to this ‘New Theory about Light’ have been published in the ‘Open Source Framework (OSF)’:

<https://osf.io/gbn4p/>

DOI: 10.31219/osf.io/gbn4p

(<https://doi.org/10.31219/osf.io/gbn4p>)

(Calculations in Mathematica 11.0)’, Page 1 – 33)

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