



330KV NIGERIA ELECTRIC POWER SYSTEM OPTIMISATION BY PRIMAL-DUAL INTERIOR-POINT TECHNIQUE FOR TWO DIFFERENT POWER POOLS.

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ABSTRACT

Optimization technique that is applied in 330 kV and other Extra High Voltage Networks are many but, all in all Primal-Dual Interior-Point (PD-IP) technique currently proves most successful. . The Technique solves load flows which are non-linear with both equality and inequality constraints at the same time thereby saving time and saving the system from encountering problems due to delays in faults clearing. The existing solves one constraint after the other and has

more than six (6) iterations before converging, while the developed method converges after first iteration. The developed technique guarantees higher system power generation with, higher loading and higher system stability. With these advantages PD-IP is most cherished and it is obtained by applying the non-negative Primal Variables, “S” and “z” into the problem formulation to transform the Inequality constraint part to Equality constraints and subsequently apply another non-negative Dual Variables, “ π ” and “v” together with Lagrange multiplier “ λ ” to solve optimisation. Optimisation is solved by incorporating, Barrier Parameter “ μ ” which ensures feasible point(s) exist(s) within the feasible region (INTERIOR POINT), Damping Factor or Step length parameter “ α ”, Step Size ΔY , in conjunction with Safety Factor “ γ ” (which improves convergence and keeps the non-negative variables strictly positive) are used for updating variables ($Y^1=Y^0+\alpha\Delta Y^0$). If initialised variables fail convergence test, iteration starts with the updated variables. The problem formulation is done

economically through minimisation of cost of power generation; $\min C(PG) = \alpha + \beta PG + \gamma PG^2$, $g(x) = 0$, stands for conventional power flow equation and other equality constraints, which is represented as; $PG - PD - \text{loss} = 0$ and $\underline{h} \leq h(x) \leq \hat{h}$, stands for operating limits on the system, which is represented as $PG_{\min} \leq PG \leq PG_{\max}$. The numerical algorithms of the method runs; Step Zero (Initialisation), Step One (Compute Newton Direction ΔY), Step Two (Update Variables), Step Three (Test for Convergence). Studies with results and analysis of improved performance by using PD-IP technique on the 330KV Bus are discussed with higher system stability obtained.

INTRODUCTION

The first known Interior-Point (IP) method is usually attributed to Frisch, which is a logarithmic barrier method that was in 1960s extensively studied by Fiacco and Mc Cormick to solve non-linear inequality constrained problems (Torren and Quintina, 2001, Granville, 2007). The greatest break-through in IP research took place in 1984, when Karmarka came up with a new IP method for Linear Programming LP reporting solution times of up to 50 times faster than the simplex method. Then Karmarka's algorithm is based on non-linear projective transformations. Later, several variants of.

Related Work

2.1 Optimisation Based On Economic Operation of Power System

Power system is operated as to supply all the (complex) loads at minimum cost. Often total load is less than the available generation capacity (Fliscounakis et al, 2013) and so there are many possible generation assignment, but when there is peak load/demand for power, it means, all the available generation capacity is used resulting in no option. During options, power generation (PG_i) is picked to minimise cost of production while satisfying load and the losses in the transmission system (Capitanescu et al, 2012) $\min C(PG) = \alpha + \beta PG + \gamma PG^2$. Also, it is noted that, small variations in demand are taken care of by adjusting the generations already on line, while large variations are accommodated basically by starting up generator units when the loads are on the upswing and shutting down when the loads decrease (Mao and Irvani, 2014). Although the problem is complicated by considering the long lead time required (6-8 hours) for preparing a "cold thermal unit for service", (Colombo and Grothey, 2013). To avoid the cost of start-up or shut-down, there is a requirement that enough spare generation capacity (spinning reserve) be available on-line in the event of a random generator failure (Awosope, 2003).

2.2 Optimisation Based On Minimum Mismatch Method

Generally, load flow equation of an N-bus network can be expressed as:

$$S = P + jQ = V^T I^* = V^T (YV)^* \quad (\text{Kamel et al, 2013}) \quad (2.1)$$

Where:-

“S” is the power injection vector

“I” is the current injection vector

“V” is the bus voltage vector and;

“Y” = G + jB is the system admittance matrix.

All the above quantities are complex, except P and Q which are real and imaginary parts of S. Because of non-linearity of load flow equations, several mathematical solutions exist, giving rise to non-uniqueness in the load flow calculations, with only one of the solutions with the minimum system losses and acceptable high voltages, as low voltage may correspond to unstable operation, is taken (Wu, et al 2010).

2.3 Optimisation Based On Fast Decoupled Load Flow Method

This is a modification of the Newton-Raphson (NR) technique which takes advantage of the weak coupling between the real and reactive power (Bhowmick et al, 2008) with two constant matrices used to approximate and decouple the Jacobian Matrix (Song and Cai, 2013).

2.4 Optimisation Based On Second Order Load Flow (Solf) Method

$$P_i = \sum_{j=1}^N (e_i e_j G_{ij} - e_i e_f B_{ij} + f_i f_j G_{ij} + f_i e_j B_{ij}), \quad (2.2)$$

$$Q_i = \sum_{j=1}^N (f_i f_j G_{ij} - f_i f_j B_{ij} - e_i f_j G_{ij} - e_i e_j B_{ij}),$$

$$E_i^2 = e_i^2 + f_i^2$$

Where E_i is modulus of i th bus Voltage. (2.3)

2.5 Optimisation Based On Mathematical Model Of Primal-Dual Interior-Point Technique

min $f(x)$

such that $g(x) = 0$
(2.4)

$$\underline{h} \leq h(x) \leq \hat{h}$$

$x \in \mathbb{R}^n$ is a vector of decision variable including control and non-functional dependent variable,

$f: \mathbb{R}^n \rightarrow \mathbb{R}$ is a scalar function representing the power system operation optimisation goal.

$g: \mathbb{R}^n \rightarrow \mathbb{R}^m$ is a vector function of the conventional power flow equation and other equality constraints.

$h: \mathbb{R}^n \rightarrow \mathbb{R}^p$ is a vector of functional variables with lower bound \underline{h} and upper bound \hat{h} representing the operating limits on the system.

It is assumed that $f(x)$, $g(x)$ and $h(x)$ are twice continuously differentiable. Since the above problem minimises $f(x)$ subject to $h(x) \geq 0$. The objective is to obtain a feasible point X . that attains the desired (Chiang and Grothey, 2014. Farivar and Low, 2013, Gan, et al 2015).

2.5.1 Greek Alphabets Used and Their Meanings in Primal-Dual Interior-Point Technique.

“ s ” and “ z ” (small and big zeta) **PRIMAL VARIABLES** are non-negative slack vectors, for transforming inequality constraint(s) to equality constraint(s) “**slack**” means loosely attached, “**Primal**” means **basic**.

“ π ” “ ν ” and (small letter pi, and nu) are non-negative Lagrangian vector called **DUAL VARIABLES**. They are vector multipliers incorporated with “ λ ” (**lambda**) the **lagrangian multiplier** to help **PRIMAL VARIABLES** solve the emerged equality constraints for optimisation.”**Dual**” means joint action.

“ μ ”(small letter mu) is a Barrier parameter or Complimentary Gap which is incorporated to ensure.

that the feasible point(s) exist(s) within the Feasible region (**INTERIOR-POINT**).

“ Ω ”(Omega) is centering parameter used with “ ρ ” (**Rho**), the confining parameter in computing “ μ ”

“ γ ”(gamma) is **safety factor** that ensures, next point satisfies positivity condition, used in computing step lengths (**damping factor**) “ α ” that improve convergence and keep non-

negative variables strictly positive. The constants “ γ ” and “ Ω ” stand for personnelemolument in the system

s, z, π and v are variables for static var compensators and FACTS (flexible ac transmission system).

Methodology

3.1 Transforming Inequality Constraint To Equality Constraints

Transformation of (2.4) is done (Yang, et al 2016) by incorporating non- negative slack vectors ‘s’ and ‘z’ into the inequality constraint $\underline{h} \leq h(x) \leq \hat{h}$, imposing strict positivity conditions on those slacks by incorporating them into logarithmic barrier terms as follows;

$$\begin{aligned}
 & \text{Min } f(x) \\
 & \text{Subject to } g(x) = 0 \\
 & -s - z + \hat{h} - h = 0 \\
 & -h(x) - z + \hat{h} = 0 \\
 & \text{Into logarithmic barrier term as} \\
 & \text{Min } f(x) - \mu^k \sum_{i=0}^p (in si + in zi) \tag{3.1}
 \end{aligned}$$

$$\begin{aligned}
 & \text{Subject to } g(x) = 0 \\
 & -s - z + \hat{h} - h = 0 \\
 & -h(x) - z + \hat{h} = 0 \\
 & “s” \geq 0; “z” \geq 0
 \end{aligned}$$

Where, k is the iteration count or number and p the number of interconnected systems for one pool.. Solving these equality constraints (Wu, et al 2012, and Ling, 2007), we apply vectors of lagrangian multipliers called Dual-Variables “ λ ” “ π ” and “ v ” together with the Newton method

$$L\mu(y) = f(x) - \mu^k \sum_{i=1}^p (in si + in zi) - \lambda^T g x - \pi^T (s - z + h - \underline{h}) - V^T (-h(x) - z + h). \tag{3.2}$$

3.2 Optimality Conditions

A local minimiser of (3.1) is expressed in terms of stationary point of $Lx(y)$ satisfying the Karush- Kuhn Tucker (KKT) optimality conditions for the NLP problem (2.4) (Torren and Quintana, 2001) as

$$\nabla y_1(y) = \left\{ \begin{array}{l} s \pi \\ z v \\ s + z - \hat{h} + \underline{h} \\ h(x) + z - \hat{h} \\ \nabla x f(x) - Jg(x)^T \lambda + Jh(x)^T v - g(x) \end{array} \right\} = 0 \quad (3.3)$$

$\hat{v} = v + \pi$ for simplification

Where l or L is local minimizer

Strict feasibility starting point is not mandatory for Primal Dual Interior Point technique but the condition $(s, z) > 0$ and $(\pi, v) > 0$ must be satisfied at every point in order to define the barrier term (Capitanescu and Wehenkel, 2012). The algorithm terminates when the Primal and Dual infeasibilities and the complementary gap fall below pre-determined tolerance otherwise, with $(s, z) > 0$ and $(\pi, v) > 0$ a new estimate y^k is computed using one step of Newton method to find zeroes (the roots) of the NL functions.

3.3 Estimating New Point (Y^k)

3.3.1 Computing Newton Direction or Step Size ΔY

The Newton direction is obtained by solving. Newton method (Tinney and Hart, 2007) with large sparse coefficient matrix (Geletu et al, 2011), with step size column matrix as below (Molzahn et al, 2013):-

$$\left\{ \begin{array}{cccccc} \pi & 0 & s & 0 & 0 & 0 \\ 0 & v & z & z & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & Jh & 0 \\ 0 & 0 & 0 & Jh^T & \nabla_x^2 l \mu & -Jg^T \\ 0 & 0 & 0 & 0 & -Jg & 0 \end{array} \right\} \left\{ \begin{array}{l} \Delta s \\ \Delta z \\ \Delta \pi \\ \Delta v \\ \Delta x \\ \Delta \lambda \end{array} \right\} = \left\{ \begin{array}{l} r_s \\ r_z \\ r_\pi \\ r_v \\ r_x \\ r_\lambda \end{array} \right\} \quad (3.5)$$

(Tinney and Hart, 2007)

$$\begin{aligned}
rs &= -s\pi + \mu^k e \\
rz &= -z\hat{v} + \mu^k e \\
r\pi &= -s - z + \hat{h} - \underline{h} \\
rv &= -h(x) - z + \hat{h} \\
rx &= -\nabla_x f(x) + Jg(x)^T \lambda - Jh(x)^T v \\
r\lambda &= g(x)
\end{aligned} \tag{3.6}$$

Where:

$$\begin{aligned}
\text{Where, } \nabla_x^2 l\mu \text{ is the combination of Hessians of objective and constraints functions. } \nabla_x^2 l\mu(y) \\
= \nabla_x^2 f(x) - \nabla_x^2 g_j(x)\lambda_j + \nabla_x^2 h_j(x) v_j
\end{aligned} \tag{3.7}$$

Where “l” is local minimiser a function of differentiation, $\nabla_x^2 f(x)$ is the Hessian or 2nd differentiation of objective function w.r.t.x, $\nabla_x^2 g(x)$ is the Hessian or 2nd differentiation of equality constraint function w.r.t.x, $\nabla_x^2 h(x)$ is the Hessian or 2nd differentiation of inequality constraint function w.r.t. x, $\nabla_x f(x)$ is the first differentiation of objective function w.r.t.x, $Jg(x)$ is the first differentiation or Jacobian value of equality constraint w.r.t.x. $Jh(x)$ is the first differentiation or Jacobian value of inequality constraint w.r.t.x. In the computation of ΔY , factorisation of the coefficient matrix (3.5) is much more expensive.

3.3.2 Computing Step Length Parameter (α)

The scalars $\alpha_P^k \in (0,1)$ and $\alpha_D^k \in (0,1)$ are step length parameters called **damping factor**. They improve convergence and keep non-negative variables strictly positive. k is the iteration counts.

$$\left. \begin{aligned}
\alpha_P^k &= \min [1, \gamma \min \{-s_i^k / \Delta s_i / \Delta s_i < 0, -z_i^k / \Delta z_i / \Delta z_i < 0\}] \\
\alpha_D^k &= \min [1, \gamma \min \{-\pi_i^k / \Delta \pi_i / \Delta \pi_i < 0, -v_i^k / \Delta v_i / \Delta v_i < 0\}]
\end{aligned} \right\} \tag{3.8}$$

The scalar $\gamma (0,1)$ is a **safety factor** which ensures that the next point will satisfy the strict positivity conditions; typical constant values, $\gamma^0 = 0.25$. $\gamma^k = 0.99995$.

3.3.3 Updating Variables

3.3.3.1 Updating control variable(s) and primal variables

$$\left. \begin{aligned} X_1^k &= X_1^{k-1} + \alpha_p^k \Delta X_1^{k-1} && \text{2 control} \\ X_2^k &= X_2^{k-1} + \alpha_p^k \Delta X_2^{k-1} && \text{variables} \\ S^k &= S^{k-1} + \alpha_p^k \Delta S^{k-1} \\ Z^k &= Z^{k-1} + \alpha_p^k \Delta Z^{k-1} \end{aligned} \right\} \quad (3.9)$$

3.3.3.2 Updating dual variables and lagrange multiplier

$$\begin{aligned} \pi^k &= \pi^{k-1} + \alpha_D^k \Delta \pi^{k-1} \\ V^k &= V^{k-1} + \alpha_D^k \Delta V^{k-1} \\ \lambda^k &= \lambda^{k-1} + \alpha_D^k \Delta \lambda^{k-1} \end{aligned}$$

3.4 Reducing The Barrier Parameter (M^k)

The scalar μ^k is the **barrier parameter** or **complementary gap** which ensures the feasible point X exist within the feasible region (Lage et al, 2009) and it is obtained by

$$\mu^{k+1} = \Omega^k \rho^k \quad (3.10)$$

Where Ω^k is chosen = $\max(0.99\Omega^{k-1}/2; 0.1)$ and it is called the **Centering Parameters**

With $\Omega^0 = (0.2 \text{ fixed})$ and $\mu^0 = (0.1 \text{ fixed})$

$$\rho^k = (S^k)^T \pi^k + (Z^k)^T V^k \quad (3.11)$$

μ^k is computed first, only if iteration (1) fails, then μ^1 and Y^1 is used to form iteration (2) as Y^0 and μ^0

(given) are used to form iteration (1)

3.5 Testing For Convergence

Interior-Point (IP) Iterations Are Considered Terminated Whenever

$$V_1^k = \max [\max\{h-h(x); h(x) - \hat{h}\}, \|g(x)\|_\infty],$$

$$V_2^k = \frac{\|\nabla_x f(x) - Jg(x)^T \lambda + Jh(x)^T V\|_\infty}{1 + \|x\|_2 + \|\lambda\|_2 + \|V\|_2}$$

Since $\|\lambda\|_2$ & $\|V\|_2$ are vectors of lagrangian multipliers,

they have no vector addition and so denominator reduces to $1 + \|x\|_2$

$$V_3^k = \frac{\rho^k}{1 + \|x\|_2} \quad (3.12)$$

$$V_4^k = \frac{|f(x^k) - f(x^{k-1})|}{1 - |f(x^k)|}$$

Typically, V_1^k and $V_2^k \leq \xi_1 = 10^{-4}$, or $\|g(X^k)\| < \xi_1$

V_3^k and $V_4^k \leq \xi_2 = 10^{-2}\xi_1$ (i.e. 10^{-6}), or $\|\Delta X\| < \xi_2$,

$\mu^k \leq \xi_\mu$ or $\xi_x = 10^{-12}$, is satisfied

Generally, $\xi_1 = 10^{-8}$ is chosen for quadratic functions with 2 variables.

If V_1^k , V_2^k and V_3^k are satisfied, then primal feasibility, scaled dual feasibility and complementary condition are satisfied which means that iterate K is a Karush Khun Turker (KKT) point of accuracy.

When numerical problems prevent verifying this condition, the algorithm stops as soon as feasibility of the equality constraint is achieved along with a very small fractional change in the objective value and negligible changes in the variables. The typical tolerances are $\xi_1 = 10^{-4}$, $\xi_2 = 10^{-2}\xi_1$ and $\xi_\mu = 10^{-12}$.

3.6 Primal-Dual Interior-Point Technique Numerical Algorithms

Step 0: (Initialisation)

Set $K = 0$, define μ^0 and choose a starting point Y^0 that satisfies the strict positivity conditions.

Step 1: (Compute Newton Direction)

Form the Newton System at the current point and solve for the Newton Direction.

Step 2: (Update Variables)

Compute the step lengths in the Newton direction and update the primal and dual variables.

Step 3: (Test for Convergence)

If the new point satisfies the convergence criteria, stop. Otherwise, set $K = K + 1$, update the barrier parameter μ^k and return to step 1.

3.6.1 Implementation of the Algorithms of Primal-Dual Interior-Point Technique

3.6.1.1 Step Zero (0), Choosing an initial point

Starting point needs only to meet the strict positivity conditions, IP method performs better if some initial heuristics are used, for instance, X^0 is between the upper and the lower limits of the bounded variables.

3.6.1.1.1 Initial point for two variables with quadratic inequality constraint $\underline{h} \leq h(x) \leq \hat{h}$

$$X^0 = \begin{Bmatrix} X_1 \\ X_2 \end{Bmatrix} = \hat{h} - \underline{h}$$

X_i^0 are tested by substituting them into $h(x)$ without considering constant term.

Example, if $h(x)$ is, $1 \leq X_1^2 + X_2^2 - 6x_1 - 2x_2 + 10 \leq 4$,

Heuristically picking X_1 as 5 and X_2 as 4 and substituting them into above inequality ignoring constants gives 3 equals the range (4 - 1) of the inequality.

$$X^0 = \begin{Bmatrix} 5 \\ 4 \end{Bmatrix} = 3$$

3.6.1.1.2 Initialising primal slack variables (S^0 and Z^0)

$$S^0 = \min [\max \{ \gamma^0 h^\Delta, h(X^0) - h \min \}, (1 - \gamma^0) h^\Delta]$$

$$S^0 = \min [\max \{ 0.25h^\Delta, h(X^0) - h \min \}; 0.75h^\Delta]$$

Where: $h^\Delta = h \max - h \min$

$$\gamma^0 = 0.25$$

$$1 - \gamma^0 = 0.75$$

$h(X^0)$ = values of X^0 including constant

$$Z^0 = h^\Delta - S^0$$

(3.13)

3.6.1.1.3 Initialising dual variables (π^0, V^0)

$$\pi^0 = \mu^0 (S^0)^{-1} e \text{ (e is diagonal I of matrix)}$$

$$\pi^0 = 0.1(S^0)^{-1}$$

$$V^0 = \mu^0 (Z^0)^{-1} e - \pi^0$$

(3.14)

$\lambda^0 = 0$ (since the power balance of steady state system is passive).

Convergence of the initial point is tested and if it fails then:

3.6.1.2 Step one (1), Computing Newton direction ΔY

With μ^0 defined and initial point Y^0 obtained; Newton method (3.5), is formed and Newton direction computed with (3.6) and (3.7) of (3.5)

3.6.1.2 .1 For two variables with quadratic constraint

With μ^0 defined and initial point Y^0 obtained; Newton method (3.5), formed and Newton direction computed with (3.6) and (3.7) of (3.5). Factorisation starts from row 3 of (3.5) where Δs is substituted for $-\Delta z$ and applied to row 1, then row 2, row 4 and with row 4, row 5 and row 6 are factorised and finally row 6 and row 7 are simultaneously factorised to obtain ΔX_1 and ΔX_2 before backward substitutions.

3.6.1.3 Step two (2), Updating variables (Y^k) with step length parameter “ α ” (3.8) . $Y^1 = Y^0 + \alpha \Delta Y^0$

Newton direction ΔY is computed from (3.5) and variables are updated from (3.9)

3.6.1.4 Step three (3), Testing for convergence

If the new point satisfies the convergence criteria, stop. Otherwise, set $K = K + 1$, update the barrier parameter μ^k and return to step 1.

IV Result and Analysis

4.1 $\min x_1^2 + x_2^2 - 4x_1 - 8x_2 + 20$ (Nagrath and Kothari, 2010)

subject to $x_1^2 + x_2^2 - 2x_1 - 2x_2 - 2 = 0$

$1 \leq x_1^2 + x_2^2 - 6x_1 - 2x_2 + 10 \leq 4$

Step 0: Initialisation

Heuristically X_1 and X_2 are chosen as follows:

$$\hat{h} - \underline{h} = h(x)$$

$$4 - 1 = 3 = h(x)$$

To choose (X_1, X_2) to give 3 we have

$$X^0 = \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} = \begin{bmatrix} 5 \\ 4 \end{bmatrix}$$

$$s^0 = 1 \min [\max \{0.25 \times 3, 13-1\}, 0.75 \times 3]$$

$$= \min [12, 2.25]$$

$$s^0 = 2.25$$

Note after obtaining x^0 from $h(x)$ as 3, subsequent $h(x)$ includes the constant term in the inequality, which is 10 to give 13.

$$z^0 = h^A - s^0, \text{ i.e } 3 - 2.25 = 0.75$$

$$\pi = \mu(S^0)^{-1}e \text{ where } e = \text{diagonal } 1$$

$$= 0.1 (2.25)^{-1}$$

$$= 0.0444$$

$$v^0 = \mu(Z^0)^{-1}e - \pi^0$$

$$\hat{\cdot}^{\wedge} 0.1333 - 0.0444, \text{ i.e., } 0.0889$$

$$V^0 = \mu(Z^0)^{-1}e = 0.1333$$

$\lambda^0 = 0$ (for passive power balance) i.e steady state condition.

$$\mu^0 = 0.1, \Omega^0 = 0.2 \text{ (fixed), } \gamma^0 = 0.25 \text{ and other } \gamma^k = 0.99995, E = 10^{-8} \text{ other } \Omega^k = 0.1$$

$$\nabla_x f(x) = \begin{bmatrix} 2x_1 - 4 \\ 2x_2 - 8 \end{bmatrix} \quad \nabla_x^2 f(x) = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$$

$$Jg(x) = \{2x_1 - 2; 2x_2 - 2\}; \quad \nabla_x^2 g(x) = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$$

$$Jh(x) = \{2x_1 - 6; 2x_2 - 2\}; \quad \nabla_x^2 h(x) = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$$

$$\nabla_x^2 \mu^0 = \begin{bmatrix} 2.17778 & 0 \\ 0 & 2.17778 \end{bmatrix}$$

$$\text{Where } \nabla_x^2 \mu = \nabla_x^2 f(x) - \sum \lambda_j \nabla_x^2 x g_j(x) + \sum_{j=1}^p \nu_j \nabla_x^2 x h_j(x)$$

Therefore, Y^0 is initialised as

s^0	2.2500	
z^0	0.7500	
π^0	0.0444	$f(x^0) = 9$
$Y^0 = v^0 =$	0.0889	$g(x^0) = 21$
x_1^0	5.0000	$h(x^0) = 13$
x_2^0	4.0000	
λ^0	0.0000	

Testing for convergence,

$$V_1^0 = 21 > 10^{-8}$$

$$V_2^0 = 0.8585 > 10^{-8}$$

$$V_3^0 = 0.0205 > 10^{-8}$$

Not converged. (Convergence failed)

ITERATION I: With Y^0 and μ^0 known, Newton System is formed and solved as follows

$$\begin{array}{cccccccccc}
 0.0444, & 0 & 2.2500 & & 0 & 0 & 0 & 0 & \Delta s^0 & 0.0000 \\
 0 & 0.0889 & 0.7500 & 0.7500 & 0 & 0 & 0 & 0 & \Delta z^0 & 0.0000 \\
 1 & 1 & 0 & & 0 & 0 & 0 & 0 & \Delta \pi^0 & 0.0001 \\
 0 & 1 & 0 & & 0 & 4 & 6 & 0 & \Delta v^0 & = -9.7500 \\
 0 & 0 & 0 & & 4 & 2.1778 & 0 & -8 & \Delta x_1^0 & -6.3550 \\
 0 & 0 & 0 & & 6 & 0 & 2.1778 & -6 & \Delta x_2^0 & 0.5334 \\
 0 & 0 & 0 & & 0 & -8 & -6 & 0 & \Delta \lambda^0 & 21
 \end{array}$$

$$\text{From (row 3) } \Delta s^0 = -\Delta z^0$$

$$\text{From (row 1) } -0.0444\Delta z^0 + 2.25\Delta \pi^0 = 0.0001$$

$$\Delta \pi^0 = \frac{0.0001 + 0.0444\Delta z^0}{2.25}$$

$$\Delta \pi^0 = 0.01973\Delta z^0 + 0.00004$$

$$\text{From (row 2) } 0.1037\Delta z + 0.75\Delta v^0 = -0.00003$$

$$\Delta z^0 = \frac{-0.00003 - 0.75\Delta v^0}{0.1037}$$

$$\Delta z^0 = -7.2324\Delta v^0 - 0.00029$$

$$\text{in row (4) } \Delta v^0 = \frac{9.7497 + 4\Delta x_1^0 + 6\Delta x_2^0}{7.2324}$$

$$\Delta v^0 = 1.3484 + 0.5532\Delta x_1^0 + 0.8298\Delta x_2^0$$

$$\text{in row (5) } 4.3901\Delta x_1^0 + 3.3184\Delta x_2 - 8\Delta \lambda^0 = -11.7418$$

$$\text{in row (6) } 3.3184\Delta x_1^0 + 7.1554\Delta x_2^0 - 6\Delta \lambda^0 = -8.6218$$

$$\text{from row (5) } \Delta \lambda^0 = \frac{11.7478 + 4.3901\Delta x_1^0 + 3.3184\Delta x_2^0}{8}$$

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$$\text{Substituting } \Delta \lambda^0 \text{ into row (6) gives } 0.0262\Delta x_1^0 + 4.6672\Delta x_2^0 = 0.1881$$

$$\text{Row (7) is } 8\Delta x_1^0 - 6\Delta x_2^0 = 21$$

$$\text{Then } \Delta x_1^0 = -2.6665, \Delta x_2^0 = 0.0553$$

$$\Delta \lambda^0 \text{ from, row (5) } = 0.02827$$

$$\Delta \lambda^0 \text{ from, row (6) } = 0.02815$$

$$\Delta \lambda^0 \text{ mean is } 0.0282$$

$$\Delta v^0 = -0.0808$$

$$\Delta z^0 = 0.5841$$

$$\Delta s^0 = -0.5841$$

$$\Delta \pi^0 = 0.0115$$

$$\alpha_p^1 = \alpha_D^1 = 1$$

$$\Delta Y^0 = \begin{array}{r} \Delta s^0 \\ \Delta z^0 \\ \Delta \pi^0 \\ \Delta v^0 \\ \Delta x_1^0 \\ \Delta x_2^0 \\ \Delta \lambda^0 \end{array} = \begin{array}{r} -0.5841 \\ 0.5841 \\ 0.0115 \\ -0.0808 \\ -2.6665 \\ 0.0553 \\ 0.0282 \end{array}$$

$$Y^1 = \begin{array}{r} 1.6659 \\ 1.3341 \\ 0.0560 \\ 0.0081 \\ 2.3335 \\ 4.0553 \\ 0.0282 \end{array} \quad \begin{array}{l} f(x^1) = 0.1143 \\ g(x^1) = 7.1131 \\ h(x^1) = 9.7290 \end{array}$$

Testing for convergence:

$$V_1^1 = 7.1131 > 10^{-8}$$

$$V_2^1 = 0.1023 > 10^{-8}$$

$$V_3^1 = 0.0183 > 10^{-8}$$

$$V_4^1 = 7.9742 > 10^{-8}$$

Convergence fail

$$\mu^1 = \Omega^1 \rho^1 \text{ from (3.10), while } \rho^1 \text{ is from (3.11),}$$

$$= 0.1 \times 0.1041 = 0.0104$$

ITERATION 2

With Y^1 and μ^1 the next Newton System is formed, from where Newton directions are computed, and variables updated with convergence tested as

$$V_1^2 = 3.1720 > 10^{-8}$$

$$V_2^2 = 0.0900 > 10^{-8}$$

$$V_3^2 = 2.2640 \times 10^{-6} > 10^{-8}$$

$$V_4^2 = 0.1198 > 10^{-8},$$

Resulting in failure of the system to converge after iteration 2. Although this process continues until at 8 IP iteration, the system converges with $x_1 = 2.0000$ and $x_2 = 2.7721$

$$\text{Where } V_1^8 = 2.8499 \times 10^{-10} < 10^{-8}$$

5.1 DISCUSSION OF RESULTS

Generally, the work reveals that Primal-Dual IP load flow technique optimisation handles power generation dispatches of different power pools with power pool 1 optimally, generates 2.0000p.u of power, while pool 2 optimally, generates 2.7721p.u.of power for the entire system stability.

5.2 SUMMARY OF FINDINGS

- 1 Other techniques solve load flow separately on two different power pools, while PD-IP technique solves them together or at the same time. Equality constraint and Inequality constraint, while the PD-IP technique solves load flow problems containing both constraints at the same time.
- 2 Number of Iterations to convergence (solution) to load flow problems of two variables are always so many for the other techniques while PD-IP technique often converges to solution within eight iterations

5.3 CONTRIBUTIONS TO KNOWLEDGE

PD-IP technique is the only load flow solution method that solves two variables, that is two different power pools simultaneously and also problems containing equality and inequality constraints.

5.4 RECOMMENDATIONS

For the technique's merits over others, intense efforts is needed to study deeper into the technique by making it accessible to various Institutions of learning and to Electricity Industries.

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