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SOLUTION OF VELOCITY, TEMPERATURE AND CONCENTRATION BOUNDARY LAYER TO THE MHD FLOW OVER A NON-LINEAR STRETCHING SHEET BY HOMOTOPY PERTURBATION METHOD

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ABSTRACT

In this study, by means of homotopy perturbation method (HPM) an approximate solution of the magnetohydrodynamic (MHD) boundary layer flow is obtained. The main feature of the HPM is that it deforms a difficult problem into a set of problems which are easier to solve. HPM produces analytical expressions for the solution to nonlinear differential equations. The obtained analytic solution is in the form of an infinite power series. In this work, the analytical solution obtained by using only two terms from HPM solution. Comparisons with the exact solution and the solution obtained by the Pade approximants and shooting method show the high accuracy, simplicity and efficiency of this method.

KEYWORDS: Concentration, MHD flow, HPM method, Boundary layer.

1. INTRODUCTION

The study of MHD boundary layer flow of an incompressible viscous fluid due to a continuously stretching surface which often introduced in many industrial and engineering sectors. Such phenomenon arises in the processes of aerodynamic extrusion of plastic surfaces, glass-fiber production and hot rolling.[1–3]

The permeability in a porous regime is the measurement capability through which fluid can flow through the medium with an ease; increasing the magnitude of the permeability, the rate of flow is also rises for given hydraulic gradient. By applying porous technologies, we can made precision- manufactured porous metal filters, components, and devices.

MHD boundary layer flow due to stretching surface with random stochastic process has been analysed by hakeem et all.[4] The conducting boundary layer flows have applications in engineering and industrial sector such as MHD generators, MHD pumps productions, MHD power generation.

The investigation of the MHD Boundary Layer Flow due to a Nonlinear Stretching Surface in a Porous regime applying Semi Analytical tricks has discussed by K. Jabeen and his group.[5] The velocity boundary layer reduces with the rise in price of magnetic parameter (M), while reverse effects is obtained in the case of concentration and thermal boundary layer.

The best analytical approach to solve the real mathematical models are both perturbation and homotopy methods. Recently, the introduction of the HPM schemes in nonlinear models has been discussed by researcher's, engineers and scientists, because it simplifies a difficult problem into a simpler one.

performance of heat flux in a porous regime embedded with Maxwell fluid past over a vertically stretched surface due to heat absorption has discussed by N. F. M. Noora and his team.[6] It is considered that the heat flux of the flow is increases as magnitude of either heat absorption or thermal radiation is increases.

A Numerical process of analyse the MHD Flow over a Stretching Sheet due to Convective Boundary Condition is discussed by Uma Munivenkatappa.[7] The present outputs of the work are compared with the existing work, and good validity is obtained between them.

Impacts of thermal diffusion and Dufour due to unsteady MHD free convection and mass transfer past through an infinite vertical permeable surface is analysed by Md.

Hasanuzzaman.[8] The mass transfer rate rises for increase in magnitude of Soret number, suction parameter, Prandtl number and Magnetic force number but decreases for Schmidt number and Dufour number.

Slip Effects due to combined Flow and Heat Transfer of a Nanofluid on a Nonlinearly Stretching Surface applying Optimal Homotopy Asymptotic Method is discussed by Gossaye A. Adem.[9] In the recent works, suspension of nanosized solid particles is considered to be classical fluids which become a familiar process for the increase in thermal conductivity of heat transfer fluids.

Optimal Homotopy Asymptotic approach on MHD Flow of a Viscoelastic Fluid due to flow with a Stretching Sheet is considered by Mohsen Zolfaghari Moghadam.[10] Moreover, the impact of important physical variables on temperature distribution, stream function and mass transfer along the stretching surface are considered and discussed using graphs.

Hydrothermal analysis over MHD squeezing nanofluid past in parallel sheets by analytical process is discussed by Kh. Hosseinzadeh.[11] The major contributions of this work are the hydrothermal investigation due to MHD nanofluid presence of squeezing flow.

Unsteady mhd three-dimensional Casson nanofluid past a porous linear stretching surface with slip condition is considered by I.S. Oyelakin.[12] It is concluded that rise in the unsteadiness of the flow tends to reduce the momentum, thermal and nanoparticles volume fraction boundary layer. The results are satisfied with previously published works.

Homotopy Analysis of MHD Free Convective Micropolar Fluid past over a Vertical stretching Embedded in Non-Darcian Thermally-Stratified regime is analysed by Olubode Kolade Koriko.[13] The comparison of the solutions found applying analytical schemes (HAM) and MATLAB package (bvp4c) is represents and a good validity is obtained.

Reaction Effects using Optimal Homotopy Asymptotic Solution for Thermal Radiation and Chemical Reaction over Electrical MHD Jeffrey Fluid past on a Stretching surface through porous regime with Heat outs Media with Heat Source is considered by Gossaye Aliy.[14] Subsequently, impacts of governing parameters of the velocity, temperature and concentration profiles are discussed and depicted graphically.

Numerical Simulation of MHD Boundary Layer Stagnation Flow of Nanofluid due to a

Stretching Surface with Slip and Convective Boundary Conditions is studied by Dodda Ramya.[15] On the basis of observation MHD stagnation point flow of water-based nanofluids with combined effect of heat and mass transfer process inherits the impacts of slip and convective boundary conditions.

Radiation a Mass Transfer impacts on MHD Boundary Layer Flow over an Exponentially Stretching surface due to Heat Source is studied by R.L.V. Renuka Devi.[16] This work presents the analyses of the heat and mass transfer conjugate due to MHD boundary layer flow over a radiating fluid and viscous incompressible with an exponentially stretching surface.

The behaviour of mhd flow and heat transfer in the presence of heat source and chemical reaction over a flat plate is analysed by Matthew.[17] Further observation reveals that temperature profile increases rapidly with increasing heat source and chemical reaction parameters while the concentration profile decreases with increasing heat source and chemical reaction parameters.

Investigation of MHD Micropolar Fluid past on an exponentially Stretching Sheet is discussed by S. Anuradha.[18] The flow behaviour, the characteristics of various dimensionless parameters study which is used in this work on the nondimensional temperature velocity and concentration are analysed numerically on the base of graphs.

Effect of Electric Field due to MHD Flow with Heat Transfer Characteristics of Williamson Nanofluid past along a Heated Surface with Variable Thickness. OHAM Solution is discussed by Gossaye Aliy.[19] Comparison of results has been made with the existing literature, and a very good validity has been obtained.

Heat Transfer Flow Over a Heated Stretching Surface subject to act of Magnetic Field is considered by Kotagiri.[20] It is concluded the temperature profile rises for the rise in magnitudes of radiation parameter.

Now a days HPM scheme is a very useful scheme to solve nonlinear as well as linear mathematical model arises in various scientific processes. In this paper, we utilise the HPM scheme to simplify the oscillating on the base of velocity boundary layer to derive an exact analytical solution. HPM scheme is an analytical scheme for evaluating the solutions to problems on the basis of introducing a homotopy with an imbedding parameter p which is considered to be very negligible as compared to the other.

In this paper, we apply homotopy perturbation technique which is utilized to evaluating the exact analytical solution to MHD flow over a non-linear stretching surface.

2. Problem formulation

Let the MHD flow of an incompressible viscous fluid is considered to be flow over a stretching surface at y = 0. The fluid is electrically conducting in nature due to the effect of magnetic field B(x) which is applied normal to the plate. The governing equations of flow variables equations of continuity due to conservation of mass and equations of velocity due to conservation of momentum are considered as follows:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial v}{\partial y}=\nu\frac{\partial^2 u}{\partial y^2}-\frac{\sigma B^2(x)}{\rho}u-\frac{\nu u}{K} \tag{2}$$

Where u is velocity component along the x-axis and v is normal velocity component along y-axis. ρ is the fluid density, ν is the kinematic viscosity, σ is the electrical conductivity of the fluid. The variable magnetic field is expressed as

$$B(x)=B_0 x^{(n-1)/2} \tag{3}$$

The corresponding boundary conditions of the stretching surfaces is

$$u(x,0)=cx^n, v(x,0)=0, u(x,y)\to 0, y\to\infty \tag{4}$$

Introducing the following similarity variables

$$\eta=\sqrt{\frac{c(n+1)}{2\nu}}x^{\frac{n-1}{2}}y, u=cx^n f'(\eta)$$
$$v=-\sqrt{\frac{c(n+1)}{2\nu}}x^{\frac{n-1}{2}}\left[f(\eta)+\frac{n-1}{n+1}\eta f'(\eta)\right] \tag{5}$$

The resulting nondimensional system of differential equations are

$$f''' + ff'' - \beta(f')^2 - M_1 f' = 0 \tag{6}$$

With BCs are $f(0) = 0, f'(0) = 1, f'(\infty) = 0$ (7)

Where $\beta = \dfrac{2n}{n+1}, M = \dfrac{2\sigma B_0^2}{\rho c(n+1)}, K_p = \dfrac{KU_0^2}{\nu}, M^2 + \dfrac{1}{K_p} = M_1$ (8)

Here f' stands for rate of change w.r.t. η

3. Homotopy perturbation method

Now we will apply the HPM scheme in order to evaluate the solution of eq. (6). Assuming *u* = *f*, eq. (6) can be expressed as this form

$$u''' + F(u) = 0 \tag{9}$$

Where $F(u) = uu'' - \beta(u')^2 - M_1 u'$ (10)

Introducing the analytical homotopy perturbation technique[16,21], wecreate a homotopy as the form:

$$u''' - \alpha^2 u' + p[F(u) + \alpha^2 u'] = 0 \tag{11}$$

with prescribed ICs

$$u(0) = 0, u'(0) = 1, u'(\infty) \to 0 \tag{12}$$

If p vanishes then Eqn (11) came as a linear equation $u''' - \alpha^2 u' = 0$, where α is an unknown parameter is to be determined later. For unit value of p, Eqn (11) converted a real problem. Now $p \in [0,1]$ and is continuously increases in the domain. Using HPM scheme we express the solution of Eqn (11) as a polynomial in p with infinite degree i.e., a power series in p.

$$u = u_0 + pu_1 + p^2 u_2 + ... \tag{13}$$

Putting Eqn (13) into Eqn (11) and comparing the constant terms with equal powers of p, we get

$$p^0 : u''' - \alpha^2 u' = 0, u_0(0) = 0, u_0'(0) = 1, u_0'(\infty) = 0 \tag{14}$$

$$p^1 : u_1''' - \alpha^2 u_1' + u_0 u_0'' - \beta(u_0')^2 + (\alpha^2 - M)u_0' = 0 \tag{15}$$

$$u_1(0) = 0, u_1'(0) = 0, u_1'(\infty) = 0$$

The solution of equation (14) with corresponding prescribed boundary conditions is expressed as

$$u_0(\eta) = \frac{1}{\alpha}(1 - \exp(-\alpha\eta)) \tag{16}$$

From Eqn. (16) & Eqn. (15) we have

$$u_1''' - \alpha^2 u_1' = (\beta - 1)\exp(-2\alpha\eta) + (M_1 - \alpha^2 + 1)\exp(-\alpha\eta) \tag{17}$$

With corresponding ICs, we obtain corresponding solution easily:

$$u_1(\eta) = \frac{\beta - 1}{6\left(\sqrt{\frac{3M_1 + 2\beta + 1}{3}}\right)^3} + \frac{1 - \beta}{3M_1 + 2\beta + 1}\eta \times \exp\left(\sqrt{\frac{3M_1 + 2\beta + 1}{3}}\eta\right)$$
$$+ \frac{1 - \beta}{6\left(\sqrt{\frac{3M_1 + 2\beta + 1}{3}}\right)^3} \times \exp\left(-2\sqrt{\frac{3M_1 + 2\beta + 1}{3}}\eta\right), \frac{3M_1 + 2\beta + 1}{3} > 1 \tag{18}$$

Hence the velocity is expressed approximately up to 1st order approximations, we have

$$u(\eta) = u_0(\eta) + u_1(\eta)$$
$$= \sqrt{\frac{3}{3M_1 + 2\beta + 1}}\left(1 - \exp\left(-\sqrt{\frac{3M_1 + 2\beta + 1}{3}}\eta\right)\right)$$
$$+ \frac{\beta - 1}{6\left(\sqrt{\frac{3M_1 + 2\beta + 1}{3}}\right)^3} + \frac{1 - \beta}{3M_1 + 2\beta + 1}\eta \exp\left(-\sqrt{\frac{3M_1 + 2\beta + 1}{3}}\eta\right)$$
$$+ \frac{1 - \beta}{6\left(\sqrt{\frac{3M_1 + 2\beta + 1}{3}}\right)^3} \exp\left(-2\sqrt{\frac{3M_1 + 2\beta + 1}{3}}\eta\right) \tag{19}$$

$$u''(\eta) = -\sqrt{\frac{3M_1 + 2\beta + 1}{3}} \left(\exp\left(-\sqrt{\frac{3M_1 + 2\beta + 1}{3}}\eta \right) \right)$$
$$- \frac{2(1-\beta)}{\sqrt{3(3M_1 + 2\beta + 1)}} \exp\left(-\sqrt{\frac{3M_1 + 2\beta + 1}{3}}\eta \right)$$
$$+ \frac{1-\beta}{3}\eta \exp\left(-\sqrt{\frac{3M_1 + 2\beta + 1}{3}}\eta \right)$$
$$+ \frac{2(1-\beta)}{3\sqrt{\frac{3M_1 + 2\beta + 1}{3}}} \exp\left(-2\sqrt{\frac{3M_1 + 2\beta + 1}{3}}\eta \right) \quad (20)$$

Where slopes are expressed as

$$f''(0) = u''(0) = -\sqrt{\frac{3M_1 + 2\beta + 1}{3}} \quad (21)$$

From the solution of HPM scheme for

$$f''(0) = -\sqrt{M_1 + 1} \quad (22)$$

4. Table

Table 1: Comparison of the values of $f''(0)$ and shooting method obtained by HPM, Pade approximants.

β	M	Pade appxn	HPM	Shooting method
1.5	0	-1.1546	-1.154600	-1.1485
1.5	1.0	-1.5251	-1.525725	-1.5251
1.5	5.0	-2.560	-2.560123	-2.5160
1.5	10.0	-3.3662	-3.366200	-3.3662
1.5	50.0	-7.1646	-7.164600	-7.1647
1.5	100.0	-10.0775	-10.07750	-10.0664
1.5	500.0	-22.3903	-22.39030	-22.39030
1.5	1000.0	-31.6437	-31.64370	-31.64370
-1	1.0	-0.8510	-0.851000	-0.8510
-1	5.0	-2.1628	-2.162801	-2.1628
-1	10.0	-3.1100	-3.110001	-3.1100
-1	50.0	-7.0475	-7.047500	-7.047500
-1	100.0	-9.9833	-9.983300	-9.9832
-1	500.0	-22.3532	-22.35320	-22.35320
-1	1000.0	-31.6175	-31.61757	-31.61757
-1.5	1.0	-0.6531	-0.653109	-0.653109

-1.5	**5.0**	**-2.0851**	**-2.085101**	**-2.085102**
-1.5	**10.0**	**-3.0561**	**-3.056109**	**-3.056109**
-1.5	**50.0**	**-7.0237**	**-7.023701**	**-7.023701**
-1.5	**100.0**	**-9.9665**	**-9.966507**	**-9.966507**
-1.5	**500.0**	**-31.631**	**-31.63101**	**-31.63101**
-1.5	**1000.0**	**-9.9867**	**-9.986701**	**-9.986701**

Table: 1 shows the comparison of our result with previously published result for HPM, shooting technique and Pade approxns. While evaluating the velocity gradient we have consider the various magnitudes of β and M keeping other pertinent parameter of the flow phenomena as fixed.

5. CONCLUSIONS

In this paper we have utilized HPM scheme in order to solve the steady flow of a viscous fluid subject to the effect of variation of magnetic field. It is very easy to handle and very effective due to the only 1st order approximation provides as very high-class accuracy. It is a conventional and promising technique and may be very much useful in future study for various branch of engineering and science.

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