

EFFECT OF COUPLED PHASE ON CHIMERA STATES IN COUPLED NEURON NETWORKS

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Article Received on 14/03/2022

Article Revised on 04/04/2022

Article Accepted on 24/04/2022

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The chimera state in the coupled neuron oscillator system is the self-organized dynamic behaviour of synchronous and non-synchronous coexistence in space, which attracts much attention because of its close correlation with hemibrain sleep. Based on the influence of the phase angle in the rotating coupling matrix, based on the coupled neuron

oscillator model. It is shown that changing the phase of the rotating coupling matrix can control the attractive and repulsive interactions between neuronal fast and slow variables, thus in single cluster chimera, multi-cluster chimera, traveling wave, steady amplitude chimera, chimera death, and synchronous states. These results suggest that the rotation-coupled phase plays a key role in regulating the dynamic behaviour of neural networks, useful for the understanding and prediction of neural networks.

KEYWORDS: non-local; chimera state; neuronal network; coupled phase.

1. INTRODUCTION

The dynamical behavior of coupled oscillator systems such as full synchronization, cluster synchronization^[1], chimera state^[2], Oscillation quenching, etc^[3-5] The realization of the function of many systems is closely related to the state control. Among them, the chimera state is the phenomenon of spatially coherent states due by coupling action in a coupled non-holophase oscillator system, which is first caused by Kuramoto^[6] Found in the Ginzburg-Landau phase oscillator model.^[7] Moreover, the amplitude-based chimera states can also be observed under certain conditions^[8], That is, the amplitude of chimera states.^[9] In contrast to phase chimera states, these amplitude chimera states exist with delay coupling^[10] Or a noise-

driven system.^[11] Medium and depends on the initial conditions, belongs to the transient spot plot structure. Chimera state phenomena are later observed in many different systems, such as chemistry^[12], optics^[13], physics^[14] and biology.^[15] It is found that chimera states are closely related to the functional realization of the brain, such as the phenomenon of hemibrain sleep in dolphins with the synchronic and dissynchronic coexistence of brain neural networks.^[16] When sleeping in an unfamiliar environment, one hemisphere of the brain is more alert than another^[17] The phenomenon is also related to the chimera states. In addition, synchronization is also crucial for normal physiological functions, and abnormal synchronization is closely related to seizures.^[18] During seizures, some areas of the brain are excessively synchronized, but others are not.^[19] Parkinson's syndrome is synchronized activity in the brain state of damaged neurons.^[20-21]

In chemical and biochemical systems, attractive coupling exists in most of the processes, while repulsive couplings appear relatively little and receive less attention. However, a small amount of existing repulsive coupling also has an important impact on the functional implementation of the system. The rich kinetic phenomena arise under the competition between attractive and repulsive coupling in coupled systems. Xiao et al^[9], In the Lorentz system, adding repulsive coupling yields stable amplitude chimera states independent of the initial conditions. And as the repulsive coupling increases, the amplitude chimera state changes to the chimera death state. Repulsive coupling also plays an important role in neural networks. Jiang et al^[22]It is found that after the repulsive coupling is added to the FitzHugh Nagumo (FHN) system, the originally only synchronous and traveling wave states will change to chimera and isolated states. Sathiyadevi et al^[23] The interaction of attraction and repulsive coupling is studied in the van der pol-duffing (VDP) oscillator, allowing for the transition to multi-cluster strange death. Thus we shows that repulsive coupling has a very important effect on chimera states.

To further investigate the effect of the competition between repulsive coupling and attractive coupling interactions on the dynamical behavior of the coupled oscillator system with nonlocally coupled neuron FHN oscillators^[24-25]We study the effects of different attractive and repulsive coupling competition effects on the stability of strange states in coupled oscillators. To this end, a rotating coupling matrix is introduced to change the strength of the attraction line and the repulsive coupling by changing the value of the coupling phase angle in the coupling matrix, and the feedback strength of the neuronal oscillator. Thus a large

number of dynamical and chimera state phenomena, and the transition between these phenomena are observed in the parameter space. This suggests that the competition between attraction-repulsive coupling in coupled neural models is crucial for the generation of chimera states in coupled oscillator systems.

2. Coupling the FHN oscillator model

Using a one-dimensional non-local ring-coupled FHN neuronal oscillator system model,

$$\epsilon \frac{du_i}{dt} = u_i - \frac{u_i^3}{3} - v_i + \frac{\sigma_1}{2R} \sum_{i=j-R}^{j+R} [b_{uu}(u_j - u_i) + b_{uv}(v_j - v_i)] \quad (1a)$$

$$\frac{dv_i}{dt} = u_i + a_i + \frac{\sigma_2}{2R} \sum_{i=j-R}^{j+R} [b_{vu}(u_j - u_i) + b_{vv}(v_j - v_i)], i = 1, 2, \dots, N \quad (1b)$$

where u_i and v_i are membrane voltages and slow recovery variable of the i^{th} node of the neural system, respectively, ϵ is a small quantity greater than 0, representing the timescale ratio of the fast and slow variables, fixed $\epsilon=0.05$; a_i representing the control parameter of the system, which determines the dynamic characteristics of the oscillator in the whole system. When $|a_i| < 1$, the system is in an oscillatory state, and when $|a_i| > 1$, it is in an excited state. In this paper, we mainly consider the study of oscillatory states and so take $a_i=0.5$. $\sigma_{1,2}$ represent the coupling strength, normalizing the coupling radius $r = R/N$, which N is the total number of oscillators and $2R$ is the range of coupling action between oscillators.

Another important feature of Equation (1) is that it contains not only direct $u-u, v-v$ variables coupling, but also, cross-coupling between $u-v, v-u$ variables. In order to effectively control the strength of the coupling action between fast and slow variables and between attraction and rejection, the rotation coupling matrix is introduced.^[26]

$$B = \begin{pmatrix} b_{uu} & b_{uv} \\ b_{vu} & b_{vv} \end{pmatrix} = \begin{pmatrix} \cos \varphi & \sin \varphi \\ -\sin \varphi & \cos \varphi \end{pmatrix} \quad (2)$$

The rotation matrix is controlled by its phase φ , where $\varphi \in [-\pi, \pi]$ By changing the phase φ , the coupling strength $\sigma_{1,2}$, and the coupling range R , the coupling mode of action, the coupling action strength and the coupling action range can be regulated accordingly.

3. Simulation and results

Without loss of generality, set the total number of neurons to $N=1,000$. The initial conditions of u_i and v_i are randomly distributed over a circle of radius 2, namely: $u_i^2(0) + v_i^2(0) = 4$. The fourth order Runge-Kutta algorithm was used to numerically solve the equation by (1) with a time step of 0.01.

We first study the effects of attractive and repulsive coupling effects on the dynamics of the coupled neuronal oscillator system when $\varphi = \pi/2 - 0.1$ dominated by cross-feedback coupling of fast-slow variables. At this time, the coupling coefficient of the cross $v_j - v_i$ variable in equation (1a) is positive, corresponding σ_1 to the attractive coupling, while the coupling coefficient of the cross $u_j - u_i$ variable in the same variable coupling equation (1b) is negative, the corresponding σ_2 should be repulsive coupling, while the coupling term of the same variable has a secondary influence. For simplicity, fixing the attractive coupling strength $\sigma_1 = 0.28$ and changing the repulsive coupling strength, it shows that as the repulsive coupling strength increases from zero, the coupled oscillator system changes from synchronous to single cluster chimera to isolated, and finally returns to synchronization, as shown in the spatiotemporal spot plot in Figure 1. When only attractive coupling $\sigma_1 = 0.28$ and no repulsive coupling, the coupled neuron oscillators are in a synchronized state, Figure 1 (a). When the repulsive coupling strength increases to $\sigma_2 = 0.1$, the coupled oscillator in synchronization shows some oscillation of the synchronization, and some remain synchronized to produce a single cluster strange state, Figure 1 (c). Further increasing the repulsive coupling strength to $\sigma_2 = 0.4$, the asynchronous oscillator separates from the main coherent region and becomes some isolated points, forming chimera states composed of isolated oscillators, Figure 1 (e). When the repulsive coupling is further increased to $\sigma_2 = 1$, the isolated oscillators eventually merge into the synchronous group and make the coupled oscillators form synchronized states again, Figure 1 (g). Under the competition of repulsive coupling and attractive coupling, the change process of the oscillator system in different space-time spot maps can also be manifested by the spatial snapshot map of the membrane potential at some time ($t = 5000$), Figure 1 (b) (d) (f) (h).

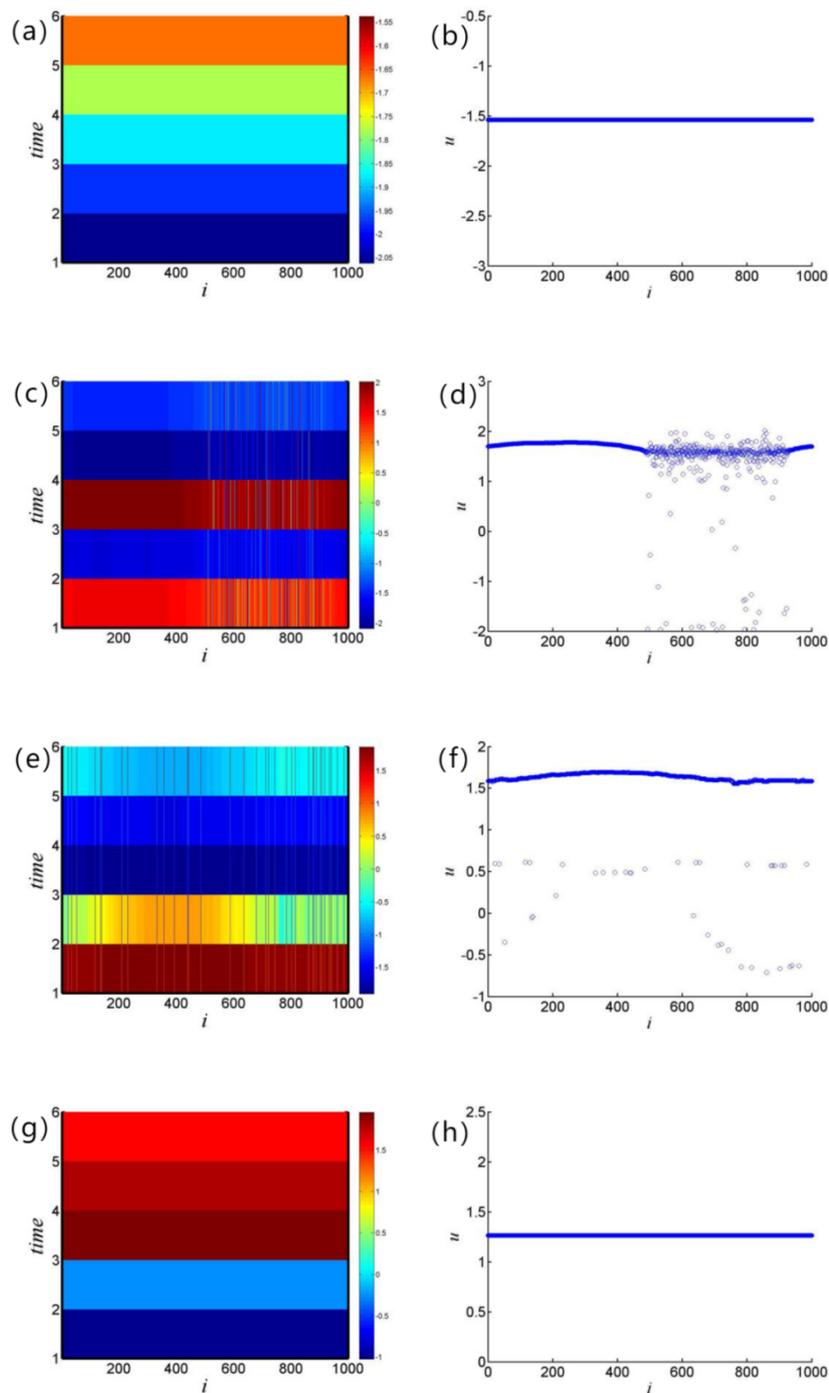


Figure 1: the spatial and temporal distribution map of the fast variables (left) and the corresponding snapshot map at time $t=5000$ (right). (a, b) synchronous, $\sigma_2=0$. (c, d) single cluster chimera, $\sigma_2=0.1$. (e, f) isolated, $\sigma_2=0.4$. (g, h) synchronized, $\sigma_2=1$. The remaining parameters are: $\varphi = \pi/2 - 0.1$, $r=0.38$, $\sigma_1=0.28$.

Next, we investigate the effect of the attractive and repulsive coupling action on the dynamics of the coupled neuron oscillon system, dominated by direct feedback coupling of fast and slow variables. Phase $\varphi = \pi - 0.1$, at this time, the direct $u_j - u_i$ variable coupling coefficient in the equation (1a) is negative, the corresponding σ_1 is positive, it is repulsive coupling, and the coupling coefficient of cross $v_j - v_i$ variable coupling term in the equation (1b) is negative, the corresponding σ_2 is repulsive coupling, negative is attractive coupling, and cross variable coupling term for secondary influence. Similarly, the repulsive coupling strength is fixed $\sigma_1 = 0.28$, and then the attractive coupling strength σ_2 is changed. When, the two direct variables are repulsive coupled, and the system is a traveling wave state, as shown in Figure 2 (a). When the coupling strength becomes negative values, the direct variable is the attractive coupling. When $\sigma_2 = -0.7$, the coupled oscillator system changes from stable traveling wave states to multi-cluster chimera states, as shown in Figure 2 (c). When $\sigma_2 = -1$, the coupled system once again transitioned to an amplitude chimera state, as shown in Figure 2 (e). In contrast to phase chimera states, amplitude chimera states refer to the existence of the amplitude of the oscillator. When $\sigma_2 = -1.5$, the coupled oscillator system further transitions to the chimera dead state, as shown in Figure 2 (f), where the coupled oscillator dies in oscillation, alternating two regions of spatially coherent and spatially incoherent states. The change process of the spatiotemporal spot map of the oscillator system in different states can also be shown by a spatial snapshot of the membrane potential u_i at some time ($t = 5000$), Figure 2 (b) (d) (f) (h).

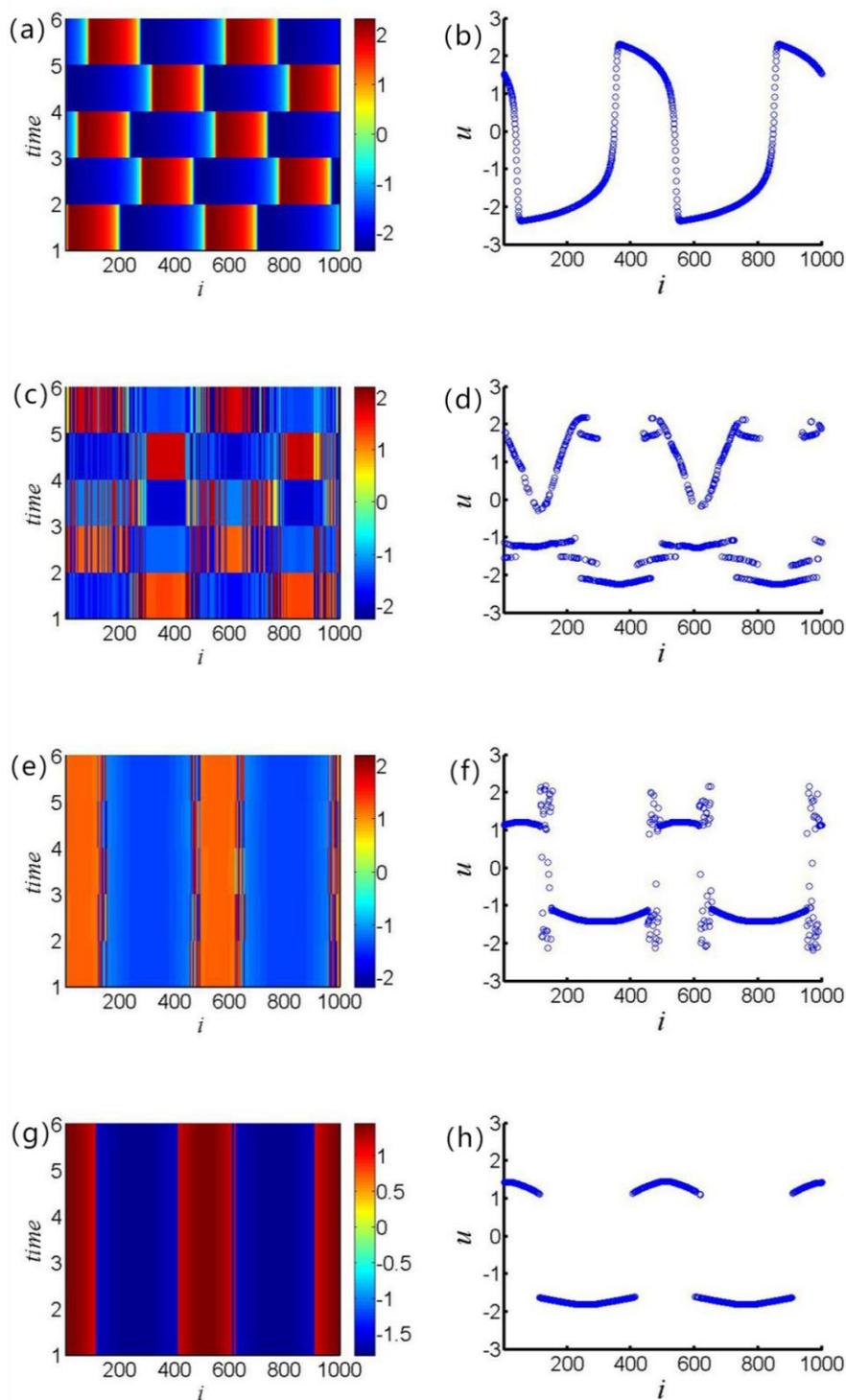


Figure 2: Spatiotemporal distribution diagram of fast variables with parameters of $\varphi = \pi - 0.1, r = 0.38, \sigma_1 = 0.28$ (left) and the corresponding snapshot map at time = 5000 (right). (a, b) traveling wave state, $\sigma_2 = 0.5$. (c, d) Multiple-cluster chimera states, $\sigma_2 = -0.7$. (e, f) The amplitude of the chimera states, $\sigma_2 = -1$. (g, h) chimera death state, $\sigma_2 = -1.5$.

Based on the phase values of two specific coupling matrices, the coupled oscillator system exhibits different patch structures by varying the strength of the attraction versus repulsive coupling action. For a given coupling strength, changing the phase values of the coupling matrix can similarly place the coupled oscillator system in different patch map structures. To further explore the effect of the phase on the FHN neural network, fix the two coupling strengths $\sigma_1=0.28$, $\sigma_2=-1$. When phase $\varphi=\pi-0.1$, magnitude chimera states appear, as shown in Figure 3 (a). As the phase decreases to $\varphi=15\pi/16-0.1$, the amplitude chimera state changes to chimera death states, as shown in Figure 3 (c). Continuing by decreasing the phase to $\varphi=-0.1$, an isolated state appears, as shown in Figure 3 (e). That is, states where only one coherent and incoherent regions coexist. When the phase decreases to $\varphi=-3\pi/4-0.1$, we find chimera states of multiple clusters, namely, states where multiple coherent and incoherent regions coexist, as shown in Figure 3 (g). When the phase decreases to very small, near the $-\pi$ boundary, $\varphi=-15\pi/16-0.1$, we find traveling wave states, as shown in Figure 3 (i). The process of change in the spatiotemporal spot map of the oscillator system in different states can also be shown by a spatial snapshot of the membrane potential u_i at some time ($t=5000$), Figure 3 (b) (d) (f) (h) (j).

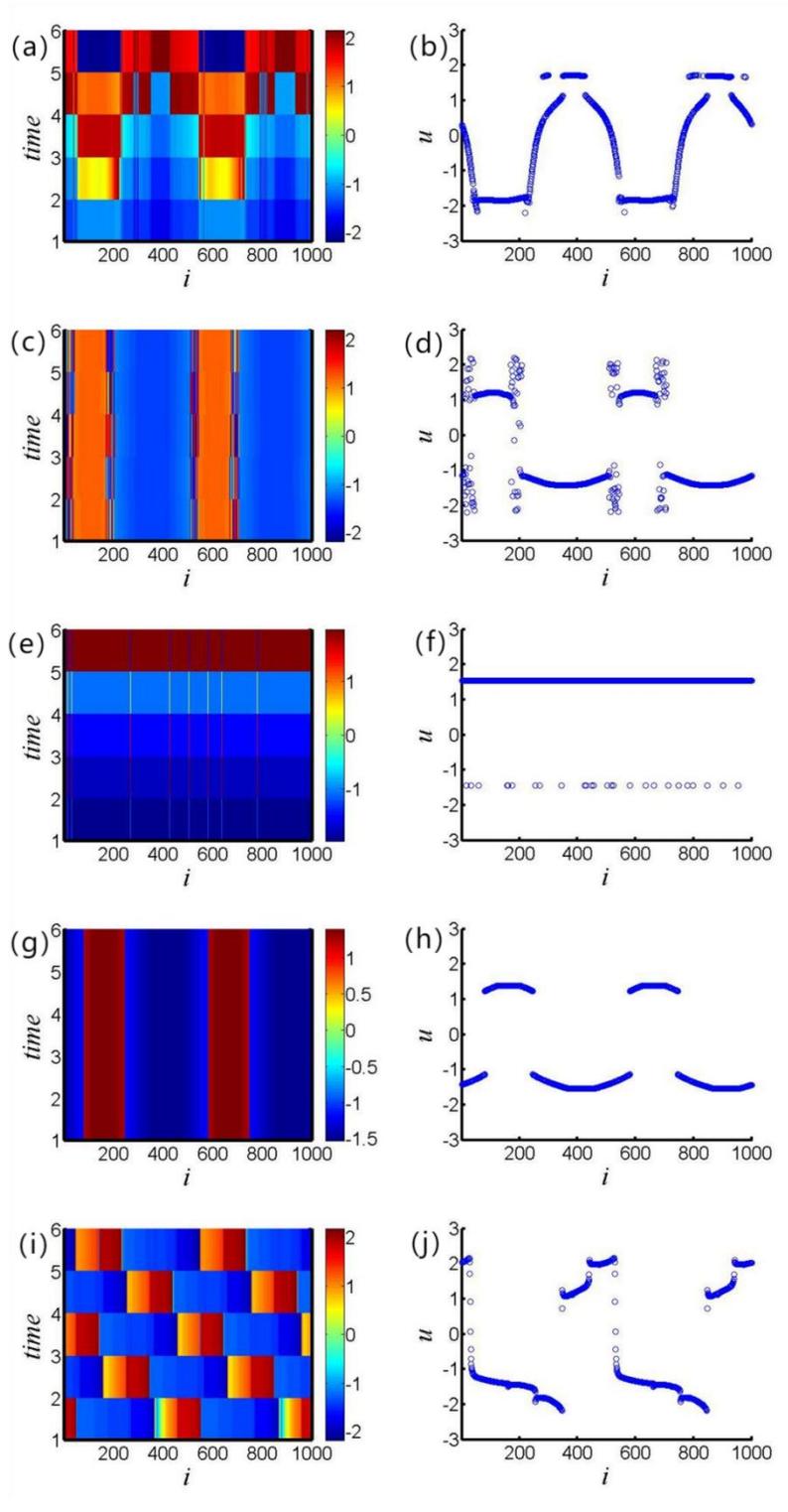


Figure 3: Spatiotemporal distribution map of the fast variables (left) and the corresponding spatial distribution map at time $t=5000$ (right). Effect of phase: Changing the phase changes the coupling of the fast and slow variables between the systems, resulting in different strange state behaviors. (a, b) amplitude chimera state, $\varphi = \pi - 0.1$ (c, d) chimera death state, $\varphi = 15\pi/16 - 0.1$. (e, f) Isolation, $\varphi = -0.1$. (g, h) Multicluster chimera states, $\varphi = 15\pi/16 - 0.1$. (i, j) amplitude chimera state, $\varphi = \pi - 0.1$

$\varphi = -3\pi/4 - 0.1$ (i, j) traveling wave state, $\varphi = -15\pi/16 - 0.1$. The remaining parameters are, $r=0.38$, $\sigma_1=0.28$.

In order to fully understand the coupling matrix phase and attraction in neuralnetwork, figure 4 shows the parameter space (φ, σ_2) phase diagram, different coupling strength σ_2 and phase φ can be very rich dynamics: traveling wave state (black square), multiple cluster (red circle), single cluster (blue triangle), amplitude strange state (pink inverted triangle), strange death state (green diamond), synchronization (dark blue hexagonal), overflow, chaos, moving chimera (purple pentagram region).

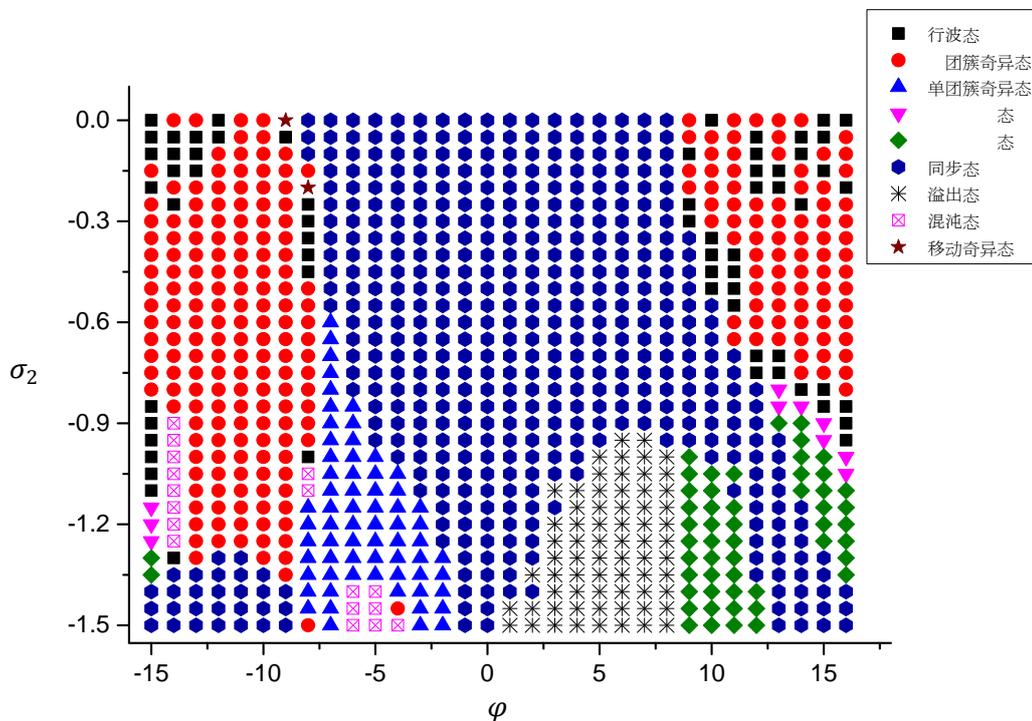


Figure 4: Phase diagram of the different kinetic behaviors in the (φ, σ_2) plane in a non-local attraction-repulsive coupled FHN cell network. There are nine main modes in space: traveling wave state (■ region), multi-cluster chimera state (● region), single cluster chimera state (▲ region), amplitude chimera state (▼ region), chimera death state (◆ region), synchronous state (⬤ region), overflow state (* region), chaotic state (⊠ region), and mobile chimera state (★ region). The parameter was set $r=0.38$, $\sigma_1=0.28$.

4. DISCUSSIONS

The coupled mode of action of the coupled neuronal oscillators has a significant effect on the patch map structure of the coupled oscillators. When the rotational coupling mode is adopted, the phase of the coupling matrix is changed, changing the coupling mode of action accordingly. When the phase is near $\pi/2$, the cross-coupling between the coupled neuron oscillators dominates, and one is attractive coupling and the other is repulsive coupling. Under the competition of attraction and repulsive coupling, as the strength of repulsive coupling increases, the coupled neuron oscillator system will transition from synchronous state to single-cluster chimera state, then to isolated state, and finally return to synchronization state. However, when the phase is nearby π the direct coupling dominates, similarly, by changing the positive and negative of the coupling strength to realize the attractive and repulsive coupling competition, resulting the coupled neuron oscillator from the original pure repulsive coupling to the amplitude chimera and chimera death state. Based on the phase diagram of the parameter space (φ, σ_2) , we know that the phase and coupling strength in the coupling matrix are closely related to the dynamical patch map structure of the coupled neuronal oscillator network. Changing the phase of the coupling matrix can change the strong and weak relationship between attraction and repulsive coupling, and the coupling mode between fast and slow variables, and then realize the interconversion between traveling wave, synchronous, amplitude chimera and death chimera states. Since the spot map structure of coupled neuronal oscillators is related to brain function realization and disease, we hope that the analysis results of phase influence on the spot map structure will help to understand the production conditions and mechanisms of chimera states in neural networks to help handle the occurrence of certain neurological diseases.

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