

NINETY-SIX PHASE CODE DESIGN WITH GOOD MERIT FACTOR

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ABSTRACT

Sequences with good Merit Factor (MF) are useful for pulse compression radar and communication applications. In this paper Ninety-Six Phase sequences are synthesized using Modified Genetic Algorithm (MGA). Modified Genetic Algorithm is used as a statistical technique for obtaining approximate solutions to combinatorial

optimization problems. This algorithm combines the good methodologies of the two algorithms like global minimum converging property of Genetic Algorithm (GA) and fast convergence rate of Hamming scan algorithm. The synthesized sequences have merit factor better than well-known Frank codes. The synthesized sequences also have complex signal structure which is difficult to detect and analyze by enemy electronics support measure.

KEYWORDS: Autocorrelation, Merit factor, Hamming scan, Polyphase codes, Genetic algorithm, Radar signal.

I INTRODUCTION

Sequences with low aperiodic autocorrelation sidelobe levels are useful for channel estimation, radar, and spread spectrum communication applications. Sequences achieving the minimum peak aperiodic autocorrelation sidelobe level one are called Barker Sequences.

The aperiodic autocorrelation function (ACF) of sequence S of length N is given by,

$$A(k) = \begin{cases} \sum_{n=0}^{N-k-1} s_n s_{n+k}^* ; & 0 \leq k \leq N-1 \\ \sum_{n=0}^{N+k-1} s_n s_{n-k}^* ; & -N+1 \leq k \leq 0 \end{cases} \quad \dots(1)$$

If all the sidelobes of the ACF of any polyphase sequence are bounded by

$$|A(k)| \leq 1, 1 \leq |k| \leq N-1 \quad \dots (2)$$

Then the sequence is called a generalized Barker sequence or a polyphase Barker sequence. In 1953,^[1] Barker introduced binary sequences for lengths $N = 2, 3, 4, 5, 7, 11$, and 13 , fulfilling the condition in eq. (2). The binary Barker can be regarded as a special case of polyphase Barker sequences. Binary codes that yield minimum peak sidelobes but do not meet the Barker condition are often called Minimum Peak Sidelobe (MPS) codes.^[2] If the sequence elements are taken from an alphabet of size M , consisting of the M^{th} roots of unity.

$$S_m = \exp\left\{2\pi i \frac{m}{M}\right\} =: \exp(i\phi_m) \quad 0 \leq m \leq M-1 \quad \dots (3)$$

The sequence is alternatively named an M -phase Barker sequence. In 1965, Golomb and Scholtz,^[3] first investigated generalized Barker sequences and presented six phases Barker sequence of lengths $N \leq 13$. Recently, in,^[4] polyphase Barker sequences of lengths $46-63$ were presented, wherein, an alphabet size of 2000 had to be used. However, polyphase Barker sequences for larger lengths require larger alphabets and the possibility for exhaustive search diminishes. The other well-known polyphase codes with ideal autocorrelation are Frank codes.^[5] Frank codes exist in perfect square length only. The synthesis of polyphase codes with good correlation properties is a nonlinear multivariable optimization problem, which is usually difficult to tackle. The Genetic Algorithm (GA) technique proved to be an efficient and powerful tool to find optimal or near optimal solutions for complex multivariable nonlinear functions but has slow convergence rate. The concept of Hamming scan algorithm has been employed for obtaining the pulse compression sequences at larger lengths with good correlation properties.^[6,7] This algorithm has fast convergence rate but has demerit the viz., the tendency to be stuck with local minima. The MGA has global minimum estimation capability of GA algorithm and fast convergence rate of Hamming scan algorithm.^[6,7,8] Binary code is one of the most commonly used radar pulse compression signals due to the easy signal generation and processing.^[2,9,10] Polyphase signal has larger main lobe-to-peak sidelobe ratio over binary signal of the same code length. In addition, polyphase waveforms have a more complicated signal structure and thus, are more difficult to detect and analyze by an enemy's electronic support measures (ESMs). With the maturity of digital signal processing, the generation and processing of polyphase signals has become easy and less costly. Therefore, polyphase code is increasingly becoming a favorable alternative to

the traditional binary code for radar signals and can be used as the basic code for radar signal design. In this paper, MGA has been used for the design of Ninety-Six Phase sequences with good correlation properties.

II. NINETY SIX PHASE SEQUENCES DESIGN

The Ninety-Six Phase sequence of length N bits is represented by a complex number sequence

$$\{s(n) = e^{j\phi_m(n)}, \quad n=1, 2, \dots, N\} \quad \dots \quad (4)$$

Where $\phi_m(n)$ is the phase of n th bit in the sequence and lies between 0 and 2π . If the number of the distinct phases available to be chosen for each bit in a code sequence is M , the phase for the bit can only be selected from the following admissible values:

$$\begin{aligned} \phi_m(n) \in \left\{ 0, \frac{2\pi}{M}, 2\frac{2\pi}{M}, \dots, (M-1)\frac{2\pi}{M} \right\} \quad \dots \quad (5) \\ = \{\psi_1, \psi_2, \dots, \psi_M\} \end{aligned}$$

For example if $M = 4$, then values of $\{\psi_1, \psi_2, \psi_3$ and $\psi_4\}$ will be $0, \pi/2, \pi$ and $3\pi/2$ respectively.

Considering a Ninety-Six Phase sequence S with code length N , one can concisely represent the phase values of S with the following 1 by N phase matrix:

$$S = [\phi_m(1), \phi_m(2), \phi_m(3), \dots, \phi_m(N)] \quad \dots \quad (6)$$

Where all the elements in the matrix can only be chosen from the phase set in eq. (5).

A more practical approach to design Ninety-Six Phase sequences with properties in eq. (2) is to numerically search the best Ninety six phase sequences by minimizing a cost function that measures the degree to which a specific result meets the design requirements. For the design of Ninety-Six Phase sequences used in radar and communication the cost function is based on the sum of square of autocorrelation side lobe peaks. Hence, from eq. (1) the cost function can be written as,

$$E = \sum_{k=1}^{N-1} |A(k)|^2 \quad \dots \quad (7)$$

The minimization of cost function in eq. (7) generates a Ninety-Six Phase sequences that are automatically constrained by eq. (2). In this optimization we have minimize the

autocorrelation sidelobe energy.

III. Merit Factor (MF)

Golay defined the merit factor MF as the ratio of main lobe energy to sidelobe energy of Autocorrelation (AC) function of sequence S. The MF, mathematically is defined as follows.^[11]

$$MF = \frac{A(0)^2}{2 \sum_{k=1}^{N-1} |A(k)|^2} \quad \dots (8)$$

The denominator term represents the energy in the sidelobes. It is related to the L_2 norm of the sidelobes

IV. HAMMING SCAN ALGORITHM

The Hamming scan algorithm is a traditional greedy optimization algorithm, which searches in the neighborhood of the point in all directions to reduce the cost function and has fast convergence rate. This algorithm mutates element of sequence one by one. The Mutation is a term metaphorically used for a change in an element in the sequence. For example if a phase value of a Ninety-Six Phase sequence is ψ_m ($1 \leq m \leq 96$), i.e., one term in eq. (6), it is replaced with phase ψ_i , $i = 1, 2, \dots, 96$, $i \neq m$, and the cost for each ψ_i change is evaluated. If the cost is reduced due to a change in phase value, the new phase value is accepted; otherwise, the original phase value is retained. The same procedure is performed for all phase values of sequence, i.e., every term of eq. (6). This process is recursively applied to the matrix until no phase changes are made. A single mutation in a sequence results in a Hamming distance of one from the original sequence. The Hamming scan algorithm mutates all the elements in a given sequence one by one and looks at all the first order-Hamming neighbors of the given sequence. Thus, Hamming scan performs recursively local search among all the Hamming-1 neighbors of the sequence and selects the one whose objective function value is minimum.

V. GENETIC ALGORITHM (GA)

GA technique, introduced by John Holland at University of Michigan proved efficient and powerful tool to find optimal or near optimal solutions for complex multivariable nonlinear functions.^[8] The major advantage of the GA algorithm over the traditional “greedy”

optimization algorithms is the ability to avoid becoming trapped in local optima during the search process.

The genetic algorithm creates a population of solutions and applies genetic operators such as crossover and mutation to evolve the solutions in order to find the best one(s). The three most important aspects of using genetic algorithms are: (1) definition of the objective function, (2) definition and implementation of the genetic representation, and (3) definition and implementation of the genetic operators. Once these three have been defined, the generic genetic algorithm should work fairly well. But the limitation of GA is slow convergence rate. This limitation is overcome by in modified Genetic algorithm.

VI. MODIFIED GENETIC ALGORITHM (MGA)

Modified Genetic Algorithm is proposed as a statistical technique for obtaining approximate solutions to combinatorial optimization problems. The proposed algorithm is a combination of Genetic Algorithm (GA) and Hamming Scan algorithms. It combines the good methodologies of the two algorithms like global minimum converging property of GA algorithm and fast convergence rate of Hamming scan algorithm. The demerit of Hamming scan algorithm is that it gets stuck in the local minimum point because it has no way to distinguish between local minimum point and a global minimum point. Hence it is sub-optimal. The drawback in Genetic algorithm is that it has a slow convergence rate because even though it may get closer to the global minimum point, it may skip it because of the methodology it employs. The MGA overcomes these drawbacks. It is quite effective to combine GA with Hamming Scan (HSA) Algorithm. GA tends to be quite good at finding generally good global solutions, but quite inefficient at finding the last few mutations to find the absolute optimum. Hamming Scan are quite efficient at finding absolute optimum in a limited region. Alternating MGA improve the efficiency of GA while overcoming the lack of robustness of HSA. MGA are introduced as a computational analogy of adaptive systems. They are modeled loosely on the principles of the evolution via natural selection, employing a population of individuals that undergo selection in the presence of variation-inducing operators such as mutation and recombination. A fitness function is used to evaluate individuals, and reproductive success varies with fitness.

Table 1: Comparison of Merit Factor of Ninety-Six Phase synthesized sequences with Frank codes.

Sequence length(N)	MF of Frank codes	MF of 96-phase Sequences
16	8	15.79
25	11.3	15.94
36	12.96	18.42
49	16.18	19.35
64	18.02	21.04
81	21.04	24.56
100	23.09	26.48
144	28.15	34.08
256	38.23	80.12

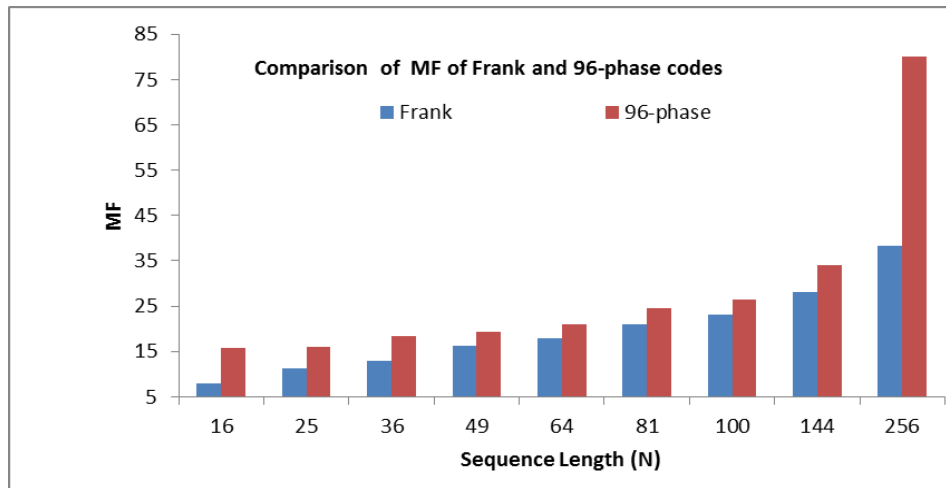


Fig. (1): Comparison of Merit factor of Frank codes and Ninety-Six Phase sequences.

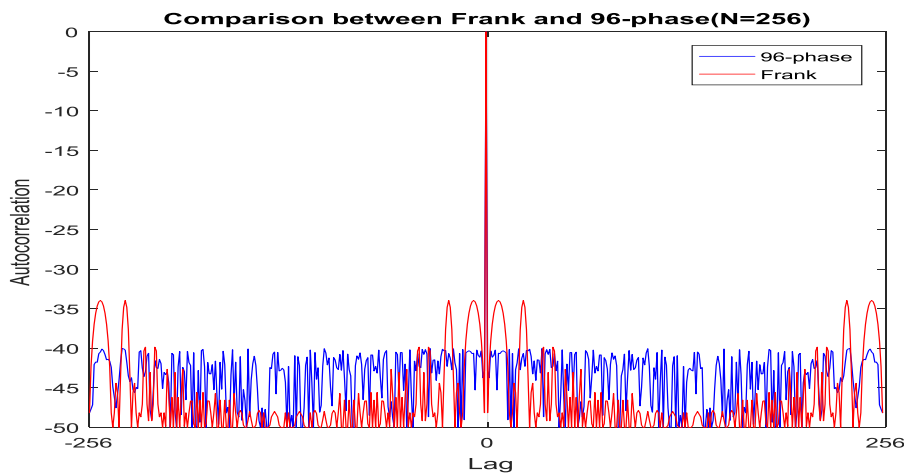


Fig. (2): Autocorrelation function of Ninety-Six phase and Frank code of length 256.

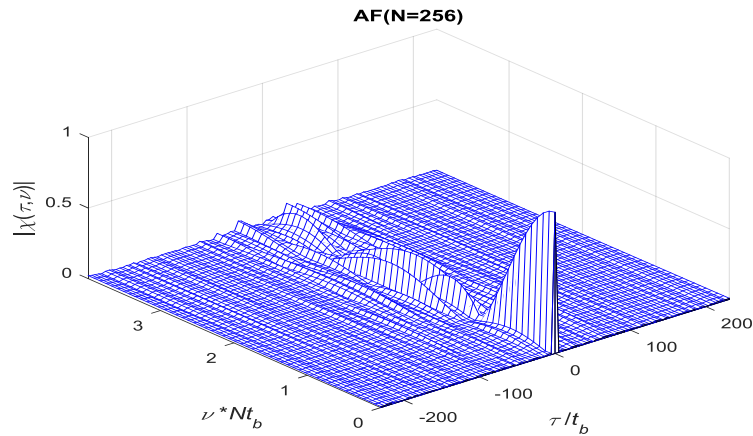


Fig. (3): Ambiguity function of Ninety-Six phase of length 256.

V. RESULTS

Ninety-Six Phase sequences are designed using the MGA, the length of the sequence, N , is varied from 16 to 289. The cost function for the optimization is based on eq. (7). Table I shows the Comparison of Merit Factors of Ninety-Six Phase synthesized sequences with Frank codes. In table I, column 1 shows sequence length, N , column 2 shows MF of Frank codes and column 3 shows the MF of Ninety-Six Phase sequences. Fig (1) shows the comparison of MF of Frank codes with synthesized Ninety-Six-phase. As shown in fig (1) the MF of Ninety-Six phase sequences are far better than well-known Frank codes. For sequence length 256 MF of Ninety six phase is more than 80 while MF of Frank code is 38.23. The synthesized sequence not only have better MF but also have a more complicated signal structure than Frank and thus, are more difficult to detect and analyze by an enemy's electronic support measures (ESMs). Fig.(2) shows the comparison of autocorrelation function of Frank code and Ninety-Six phase sequence of length 256. As shown in the figure Peak Sidelobe Level(PSL) of Ninety-Six phase sequence is 7 dB lower than Frank code of same length. Fig (3) shows Ambiguity function of Ninety-Six phase of length 256. As shown in fig ambiguity function is thumbtack.

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