

**PRACTICAL EVALUATION OF ATMOSPHERIC REFRACTION  
EFFECT ON SURVEYING OBSERVATIONS****Ismat M. Elhassan\***

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Riyadh, Saudi Arabia.**ABSTRACT**

The irregular atmospheric refraction is an important source of systematic errors in various surveying observations. One important effect is that on observed vertical angles that are needed for many surveying applications such as trigonometric levelling. In this article

the effect of atmospheric refraction on vertical angle observations is studied and results are outlined. It is observed that the highest effect of atmospheric refraction on vertical angle observation occurs during the day, when the sun is vertically above the observer. Irregular effect happens when the sun suddenly rises after being hidden by clouds.

**INTRODUCTION**

Refraction of light is defined as the bending of light as it passes from one medium into another. It is well known from the theory of optics that a ray of light changes direction as it travels from a medium of one refractive index to another medium that has a different refractive index in order to follow the quickest path. The same effect happens when a ray travels into the atmosphere, because it passes through air layers with different temperatures and hence different refractive indices.

Therefore, the refraction has an effect on the most common surveyor's measurements such as: the vertical angles (trigonometric levelling, 3D networks) as well as in the geometric levelling (Dodson and Zaher, 1985; Abdel Razek, 2020; Nikolitsas, 2021; Nikolitsas K. and Evangelia Lambrou, 2021; Lianhuan et al, 2022).

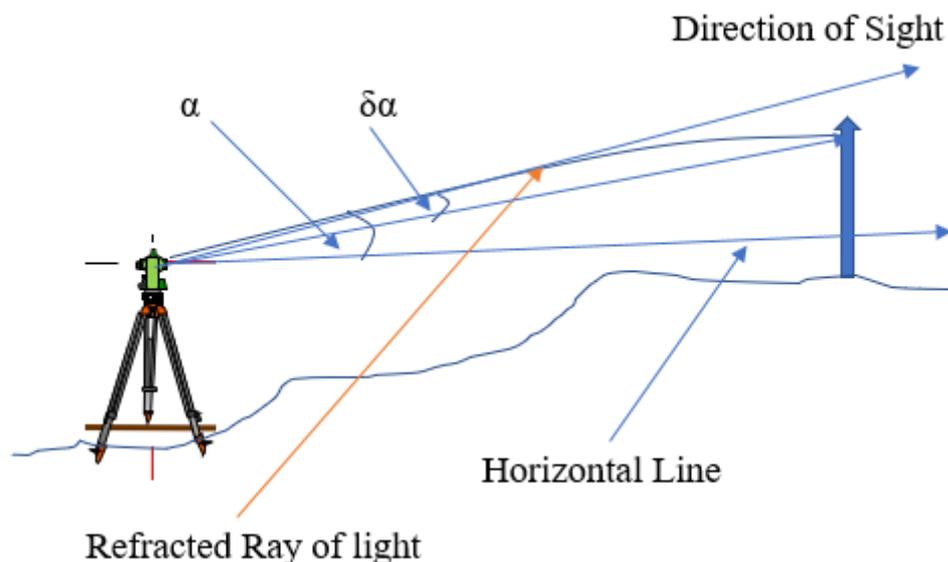
Vertical angles are usually needed in surveying operations leading to height determination or in topographic information, reduction of directly measured slope distances to horizontal and precisely determining height displacements of structures.

These vertical angles are usually observed using precise instruments of angle observation such as theodolites or total stations.

It is however noticed that observed vertical angles are affected by change in atmospheric conditions which affect the path of the light ray from target to instrument. This is known as atmospheric refraction effect which will be the subject of this research study.

### ATMOSPHERIC REFRACTION

Atmospheric refraction is the deviation of light from a straight line while passing through atmospheric layers with different air density. The air density varies as the height changes. The velocity of light changes with the change in air density, which leads to light refraction. This means that straight light path with vertical angle  $\alpha$  will be refracted by an angle  $\delta\alpha$  (Figure 1).



**Figure 1: Optical ray path from instrument to target.**

Terrestrial refraction, also known as geodetic refraction, deals with the apparent angular position and measured distance of terrestrial bodies. It is especially important during the process of object height determination and topographic mapping. The line of sight in terrestrial refraction passes near the earth's surface, which verifies that the magnitude of

refraction depends chiefly on the temperature gradient near the ground. Temperature gradient varies widely at different times of day, seasons of the year, the nature of the terrain, the state of the weather, and other factors.

As a common approximation, terrestrial refraction is considered as a constant bending of the ray of light or line of sight, in which the ray can be considered as describing a circular path. A common measure of refraction is the coefficient of refraction.

The refraction coefficient  $K$  was then defined as the ratio between a mean radius of the earth  $R$  and the light path curvature radius (Baselga, et al, 2014).

$$K = R / r \quad (1)$$

and an approximate value of  $K = 1.30$  was extensively used in geodetic height computation. The use of this approximate value for the refraction coefficient, however, is not representative of most engineering type measurements made close to the ground where heating effects predominate (Dodson and Zaher, 1985).

The use of the actual refraction coefficient is really needed for leveling and also for applying the necessary reductions of electromagnetic distance measurements (EDM) to reduce slope distances to horizontal.

There are basically three types of temperature gradient ( $dT/dh$ ).

1. Absorption: occurs mainly at night when the colder ground absorbs heat from the atmosphere. This causes the atmospheric temperature to increase with distance from the ground and  $dT/dh > 0$ .
2. Emission: occurs mainly during the day when the warmer ground emits heat into the atmosphere, resulting in a negative temperature gradient, i.e.  $dT/dh < 0$ .
3. Equilibrium: no heat transfer takes place ( $dT/dh = 0$ ) and occurs only briefly in the evening and morning.
4. The result of  $dh/dT < 0$  is to cause the light ray to be convex to the ground rather than concave as generally happens. This effect increases the closer to the ground the light ray gets and errors in the region of 5 mm/km have resulted.

The coefficient of refraction is directly related to the local vertical temperature gradient and the atmospheric temperature and pressure. The larger version of the coefficient  $k$ , measuring

the ratio of the radius of the Earth to the radius of the line of sight, is given by (Nikolitsas and Lambrou, 2021).

$$K = 503 (P/T^2) (0.0343 + dt/dh) \quad (2)$$

Where temperature T is given in Kelvin, pressure P in millibar and height h in meters. The angle of refraction increases with the coefficient of refraction and with the length of the line of sight.

There are a number of complex equations available in the literature that give reasonable approximations for the amount of angular refraction. As an example, the one by Sæmundsson (1986).

$$\delta = 1.02 * \cot \{ \Phi + [10.3 / (\Phi + 5.11)] \} * P / 101 * \{ 283 / (273 + T) \} \quad (3)$$

Where:  $\delta$  = the amount of refraction in arc minutes.

$\Phi$  = the object's true altitude in degrees.

P = atmospheric pressure in kPa.

T = temperature in Celsius.

Bennett, 1982 however, had already developed a simpler empirical formula for calculating refraction from the apparent altitude which gives the refraction  $\delta$  in arcminutes as.

$$\delta = \cot [\Phi_a + 7.31 / (\Phi_a + 4.4)] \quad (4)$$

Where  $\Phi_a$  is the apparent altitude of the astronomical body in degrees.

Nikoltsasis, 2021 tested the use of the Hydrostatic Levelling Systems (HLS) data and distance measurements with a laser tracker, it is possible to determine the refraction coefficient.

It is clear that the relation between atmospheric refraction and the temperature gradient depending on the object altitude is only necessary for refraction values to be determined when observing celestial bodies and is unnecessary for terrestrial observations.

In general, however, the correction in the vertical angle (in rad) due to refraction ( $V_{cr}$ ) can be calculated by the following equation (Nikolitsas and Lambrou, 2021),

$$V_{cr} = K D / (2 R) \quad (5)$$

Where: D = Slope distance (m); K = Refraction coefficient; R = Mean radius of the Earth m).

The effect of the atmosphere on the ray of light depends mainly on the weather condition whether it is stable or unstable.

Weather is usually unstable during the day light where density of air layers increases with the increment of elevation above the earth. During the night however, the weather condition is stable. Air layers near to the earth surface are of higher density than those away from it. Neutral state happens somewhere in between the two states.

In surveying, height differences are required for a wide range of measuring applications. Trigonometric height measurement is a well-established surveying technique for the determination of height differences. The method is however impaired by refraction effects. Refraction is detrimental to terrestrial optical measurements and can be regarded as major source of systematic error in the precise determination of distances and directions.

The above discussion indicate that there is indeed a difficulty in directly estimating K from Eq. (2). Therefore, an effort was made to establish a mathematical relationship between K and the meteorological parameters. For this, a data fit approach was implemented; the parameters included being pressure (P), temperature (T) and relative humidity (RH %). The RH is used instead of the vertical temperature gradient (dT/dH). Measuring the temperature gradient along a line neither practical nor economical in routine surveying application (Gaifiailia, et al, 2016).

Despite the progress and continuous modernization of total stations, the crucial factor in the trigonometric measurement of elevations is the impact of vertical refraction.

### **GAIFIALLIA REFRACTION COEFFICIENT MODEL**

The final selection of the derived mathematical equations shown in equations 6 to 9 was based on their relevant statistics that measure the quality of the prediction of K. Besides the linear relationship (Eq. 6), there was also an effort to define alternative models in which the variables are related non-linearly. Parameters for equations of polynomial, logarithmic, and exponential type were derived as follows (Gaifiailia et al, 2016).

Linear Model

$$K = -0.0077P - 0.0066T - 0.0024RH + 6.2414 \quad (6)$$

Polynomial 2<sup>nd</sup> order.

$$K = 0.0004(P + 0.86T + 0.32RH)^2 - 0.7182(P + 0.86T + 0.32 RH) + 287.46 \quad (7)$$

Logarithmic

$$K = -6.1343 \ln (P + 0.86T + 0.32 RH) + 41.049 \quad (8)$$

Exponential

$$K = 2^{10} 21e^{-0.0646(P+0.86T+0.32RH)} \quad (9)$$

## TEST METHODOLOGY

A test has been carried out to investigate the effect of atmospheric refraction of light on the observed vertical angle and hence determined height. An open sight within KSU has been selected to carry out the test.

A 2" electronic theodolite was used to observe vertical angles at different times on the same day. The target was set up at a station 200m away from the theodolite station.

Time of observations continued from 7:00am to 3:30pm. Vertical angle was observed every half an hour.

Using correction for vertical angle formula:  $V_{cr} = D K / 2 R$  (10)

## RESULTS AND ANALYSIS

Results of observed vertical angles, time and weather condition were recorded as in Table 1. Results of observed vertical angles versus temperature are shown on Figure 2. Computed heights are determined from the relations:

Height difference  $h_i = D (\tan \alpha_i)$ , where D is the horizontal distance between theodolite station and target;  $\alpha_i$  is the observed vertical angle.

Computed heights are shown on last column of Table 1 and are plotted against temperature in Figure 3.

**Table 1: Weather and Vertical Angle Observations on 30<sup>th</sup> of October 2020.**

Time	Temperature T °C	Pressure (P) in millibar	Pressure (P) in mmHg	Humidity HR	Observed Vertical angle in Degrees $\alpha_i'$	$h_i' = D \tan \alpha_i'$ (m)
07:00	17.0	1021	766.00	49%	1.553400	5.4237
07:30	18.0	1021	766.00	45%	1.553358	5.4245
08:00	20.0	1021	766.00	40%	1.554750	5.4283
08:30	22.0	1021	766.00	37%	1.554300	5.4273
09:00	25.0	1022	766.76	34%	1.553040	5.4216
09:30	26.0	1022	766.76	27%	1.553220	5.4234

10:00	28.0	1022	766.76	21%	1.553040	5.4224
10:30	28.5	1022	766.76	21%	1.553040	5.4225
11:00	29.0	1021	766.00	21%	1.552770	5.4216
11:30	29.0	1021	766.00	20%	1.553670	5.4244
12:00	29.0	1020	765.26	19%	1.553130	5.4225
12:30	29.5	1020	765.26	19%	1.553670	5.4240
13:00	30.0	1020	765.26	19%	1.553130	5.4225
13:30	30.0	1020	765.26	19%	1.553400	5.4230
14:00	30.0	1019	764.60	19%	1.552500	5.4210
14:30	30.0	1019	764.60	19%	1.555200	5.4300
15:00	30.0	1019	764.60	19%	1.555020	5.4290
15:30	29.0	1019	764.60	21%	1.554660	5.4281

For computation of height:  $h = D \tan \alpha$

$$dh = D (1 / \cos^2 \alpha) d\alpha$$

using equation (6) the following results were obtained for refraction coefficient:

$$K = -0.0077P - 0.0066T - 0.0024RH + 6.2414$$

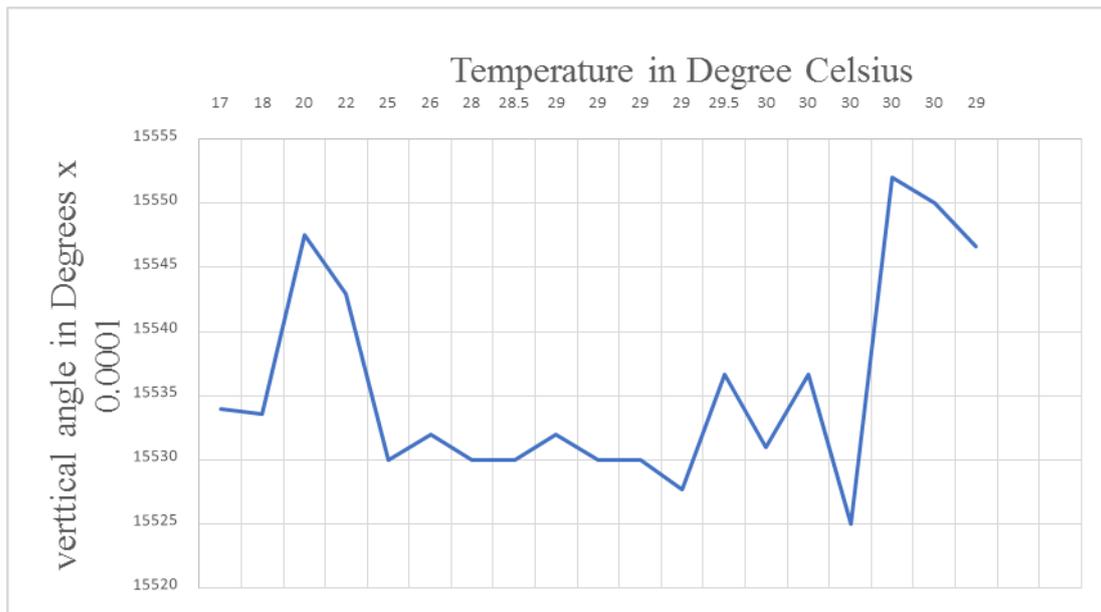
$$\text{Correction for } V \text{ Angle} = [200 K / 2 \times 6731000] \times 180 / \pi = 0.00085 K^\circ$$

**Table 2: Refraction Coefficient and Height correction.**

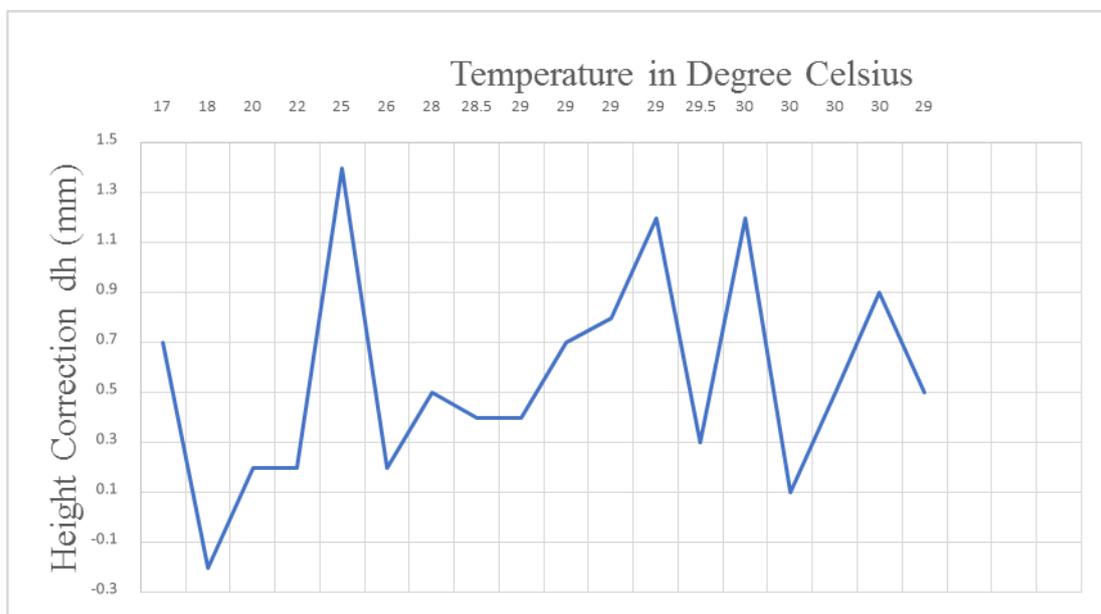
Time	Observed Vertical angle in Degrees $\alpha_i$	Refraction coefficient (K)	Corrected Vertical Angle, $\alpha_i$ in degrees	$h_i$ (m) corrected for refraction (m)	Height correction (m $\times 10^{-4}$ )
07:00	1.553400	0.231020	1.553608	5.4244	7.0
07:30	1.553358	0.224860	1.553560	5.4243	-2.0
08:00	1.554750	0.211780	1.559769	5.4285	2.0
08:30	1.554300	0.198652	1.554479	5.4275	2.0
09:00	1.553040	0.173534	1.553196	5.4230	14.0
09:30	1.553220	0.166862	1.553370	5.4236	2.0
10:00	1.553040	0.154046	1.553179	5.4229	5.0
10:30	1.553040	0.150746	1.553176	5.4229	4.0
11:00	1.552770	0.152836	1.552908	5.4220	4.0
11:30	1.553670	0.152836	1.553808	5.4251	7.0
12:00	1.553130	0.155044	1.553273	5.4233	8.0
12:30	1.553670	0.155744	1.553810	5.4252	12.0
13:00	1.553130	0.152444	1.553267	5.4223	3.0
13:30	1.553400	0.152444	1.553537	5.4242	12.0
14:00	1.552500	0.157834	1.552672	5.4211	1.0
14:30	1.555200	0.157834	1.555372	5.4305	5.0
15:00	1.555020	0.157834	1.555762	5.4299	9.0
15:30	1.554660	0.164386	1.554799	5.4286	5.0

Mean Correction =  $5.6 \times 10^{-4}$  m

= 0.56 mm



**Figure 2: Vertical angle versus Temperature**



**Figure 3: Height Correction Versus Temperature.**

## CONCOLUSIONS

The aim of this research is to analyze the effect of temperature change on the light refraction and hence observed vertical angles from which height difference is computed.

It has been observed that for temperature between 17° C and 30°C observed vertical angle and hence computed height differences were not significantly affected.

The effect of atmospheric refraction on observed height is within 0.6mm which is quite limited and not significant for most of the engineering applications.

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