wjert, 2023, Vol. 9, Issue 8, 159-174. *Review Article* ISSN 2454-695X

World Journal of Engineering Research and Technology

WJERT

www.wjert.org SJIF Impact Factor: 5.924

DERIVATION OF ORTHOMETRIC HEIGHT FROM GNSS ELLIPSOIDAL HEIGHT USING POLYNOMIAL SURFACE FITTING

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Article Received on 13/06/2023 Article Revised on 03/07/2023 Article Accepted on 23/07/2023

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ABSTRACT

The importance of geoid formation arises from the fact that it is the best way to transform ellipsoidal heights derived from Global Positioning System (GPS) measurements based on the ellipsoidal surface to orthometric heights that are normally used in surveying and civil engineering applications. There are various techniques used to form geoid models. All these are based on either gravimetric or geometric modeling. In this paper, a geometric approach where a simple polynomial model using Least squares method was developed from the combination of orthometric heights of few control points determined by automatic level observations, with ellipsoidal heights of the same points, from Differential Global Positioning System (DGPS) for a limited area within King Saud University (KSU) campus, in Riyadh, Kingdom of Saudi Arabia (KSA). Statistical test and analyses of results were carried out. It has been demonstrated that orthometric heights of ±22cm accuracy can be obtained after transforming ellipsoidal heights to orthometric heights via the formed geoid model using limited control points. The results of the obtained model compare favorably with some of the existing methods tested by other authors over larger areas using greater number of control points.

INTRODUCTION

Most of the surveying and civil engineering applications such as Digital Elevation Models, topographic mapping, route surveying and various other civil engineering projects require orthometric heights of ground points. The classical and accurate method of spirit levelling survey takes long time and requires a group of surveyors. On the other hand, Global

Navigational Satellite Systems (GNSS) receivers collect ground heights based on ellipsoid surface. Although ellipsoidal elevations are very accurately obtained, they are not applicable in many of the aforementioned operations. Hence, these ellipsoidal heights should be converted to orthometric heights. This process is done through what is called geoid modelling.

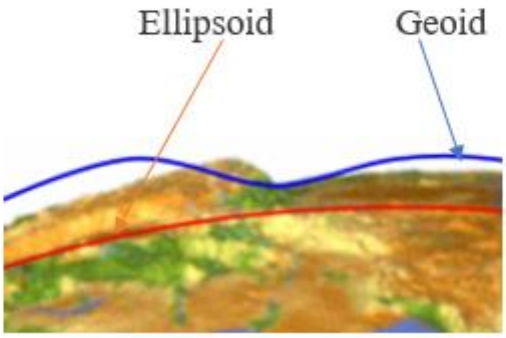

Figure 1: Ellipsoid and Geoid Surfaces.

The geoid is defined as the equipotential surface which would coincide exactly with the mean ocean surface of the earth, if the oceans were in equilibrium and extended through the continents.

The geoid surface is irregular, but considerably smoother than earth's physical surface (Figure 1). Sea level, if undisturbed by tides, currents and weather, would assume a surface equal to the geoid. Determination of the geoid has been one of major challenges of geodesists. Gravity data have been used in the past with stokes integration and least squares collocation for determining the geoid. This approach of converting GNSS ellipsoidal heights into orthometric heights is expensive and laborious. Geoid determination using integration of data from geodetic levelling and Global Navigation Satellite System (GNSS) are taking over from geoid determination using gravity data (Zhan-ji, Yang and Chen Yong-qi, 1999 and Propeller-Aero, 2021).

Data from GNSS reduces the time, cost and energy in geoid modelling. Global Positioning System (GPS) provides WGS‟84 ellipsoidal heights and when compared with orthometric height, from geodetic levelling, allows for the computation of the geoid-ellipsoid separation (N) in the region of the survey. The mathematical relation representing the geoid surface with respect to the reference ellipsoid is called the geoid model. The geometric heights obtained by GNSS and Spirit Levelling allow determination of the geoid-ellipsoid separation.

Regional geoid models may be developed by using an earth gravitational model. An accurate geoid model is substantial for the establishment of precise orthometric heights. The essential application of a geoid model is to transform GNSS heights to gravity related elevations above a local datum (Hussain, 2020).

The objective of this article is to establish a local GNSS-Leveling geoid model for a limited area in Riyadh City, capital of Saudi Arabia using geometric approach based on polynomial surface interpolation. Such model will allow transforming GNSS ellipsoidal heights to orthometric heights that can be used in various surveying and civil engineering applications.

LITERATURE REVIEW

Many researchers conducted work trying to formulate geometric geoid models. Examples of results obtained by researchers forming geometric geoid models are presented below.

Zhan-ji and Young-qi, 1999 used Kong as a test area to study various aspects of the geometric method of geoid modelling. They have found that incorporation of a geopotential model and the digital terrain model of the area of study can produce a geoid model of accuracy of ± 2–3 cm.

Lambrou, et al, 2003 attempted to make an approximation of the geoid surface over a small area of a few km^2. Two surfaces, the plane and the ellipsoid, have been tested for the best fitting via the least square method as adequate number of known points was available. The evaluation of the results indicates that a standard error of ± 0.6cm can be obtained.

Erol, B. and R. N. Çelik, 2004 formed local geoid model using IDW method and Kriging method of interpolation obtaining accuracy of ±3.42cm and ±3.07cm respectively.

Uzun and Cakir, 2006 used cubic and quadratic polynomial interpolation methods in forming geoid model and obtained RMSE values of ±9.72, ±12.00cm, respectively.

Kamguia, et al, 2007, formed a geometric geoid model for a small area in Cameroon using gravimetric approach and obtained an accuracy of ±14 cm. After a four-parameter fitting to the GPS/levelled data, the accuracy was improved to ±11 cm.

Sanlioglo, et al, 2009 formed Konya geoid model using second order-nine parameter multi-quadratic surface fitting using 1175 control points. Root mean square of model is ± 4.5 cm has been achieved.

Kaloop, 2009 compared three geometric methods for geoid undulation determination in an area in Egypt. Standard deviations of check points obtained were: ±22.0cm, ±61.0cm and ±2-3cm for multiple regression (polynomial), least square collocation and minimum curvature surface methods respectively.

Alevizakou, E. G and Evangelia Lambrou, 2011 obtained a local geoid of accuracy ±1.0cm to ±2.3cm after testing 16 network points using a plane and bilinear approximations.

Lin, 2014 generated two geometric geoid models and one gravimetric-geometric geoid model for Tainan City using the GPS and leveling data. He used fifth- and seventh-degree polynomial, and achieved orthometric height accuracy of ±2-4 cm.

Manisa, et al, 2016 obtained orthometric height of accuracy of about ±20cm when converting ellipsoidal height to orthometric height using geometric polynomial approach for an area 100km x 100km in Botswana**.**

Miky, et al, 2017 used kriging interpolation to form local geoid model. Their results indicate that the difference among estimated undulations values from the local geoid model and undulations values calculated from leveling techniques ranges from 1.8 cm to -1.1 cm.

Eteje and Olujimi, 2018 formed a local geometric geoid model whose parameters have been determined using bi-cubic model. To test the accuracy of the formed geoid model they interpolated geoid heights of eleven points. They obtained an accuracy of ± 1.7cm, implying that geoid heights can be interpolated within the application area with this accuracy.

Younis, 2019 constructed a geoid model for Palestine using finite element method. Using 2nd and 3rd order polynomial, the accuracy of ±1-3 cm has been achieved.

Fusami, et al, 2019 used Geoid Model EGM2008 and polynomial models of first, second and third orders to obtain geoid separations of accuracy ±45.7cm, ±9.4cm and ±5.7cm, respectively.

Pirti, et al, 2019, The geoid model which is obtained from a fifth order polynomial fit of the project area is good enough in this study. The discrepancy between the precise geometric and GPS levelling (with geoid corrections) is 0.8 cm over 1 km.

Bilani and Ismat, 2020 used multiple regression method was used to model the geoid in a defined area that resembles a big city. Riyadh city, capital of Saudi Arabia has been used as a case study. It has been found that fifth degree polynomials provide a model with accuracy of ±1.8 cm.

Raufu and Tata (2021) formulated geoid models for Akure city in Nigeria using three polynomial models: (DGPS) observations were carried out to determine ellipsoidal heights of the point while nine and eleven coefficients were used for the geoid and orthometric height modelling. Model A and Model C used 2-D (x, y) positions with nine and eleven parameters while model B used 3-D (x, y, Δh) positions with nine parameters. The least-squares method was adopted in computing the parameters of the models, obtaining RMSE of model A as ±14.3 cm, model B as ±15.7 cm and model C as ±14.5 cm, Chun-Jia Huang & Jen-Yu Han, 2022 demonstrated that a city scale geoid undulation model with a quality level around $\pm 14\text{mm}\sqrt{\text{km}}$ can be obtained by applying the proposed approach, meaning ±1.4cm for a 1 km^2 area.

Zaki, et al, 2022 used geometric approach to form a geoid model for Kwait and by integrating a geopotential model with a digital terrain model of the test region obtained an accuracy of ±2-±3cm.

In conclusion, previous tests achieved geometrically-obtained geoid models with accuracy ranging between 2cm to 30cm, depending on test environment, especially area size and number of control points used.

GEOMETRIC BACKGROUND

Ellipsoidal height is the vertical distance between the GNSS antenna and the Earth surface (Figure 2). Orthometric height, also known as height above mean sea level (MSL), is the vertical distance between the Earth surface and the geoid surface (Ardusimple, 2021).

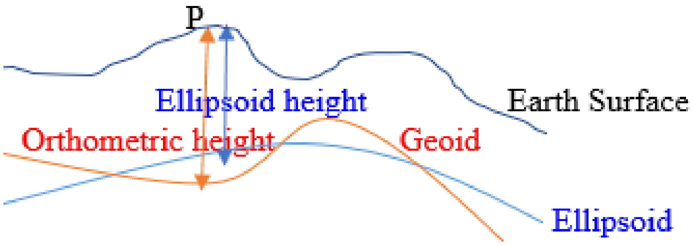

Figure 2: Ellipsoid and Orthometric heights.

The geoid height is the vertical difference between the ellipsoid and geoid at a point (Figure 3).

Figure 3: Geoid height (Geoid separation or Geoid undulation)

The height difference between the geodetic height h and the orthometric height H varies from point to point and is thus called geoid undulation, N. The key issue of transforming the GPS height into orthometric height is to determine the geoid undulation value accurately. In this paper an algorithm to develop a regional grid-based geoid model using GPS data (ellipsoidal height) and the spiritual leveling data (orthometric height) is proposed. In brief, the proposed algorithm includes the following steps

1- Establishing the functional relationship between the point's plane coordinates (such as Northing and Easting) and the undulation (N) using back-propagation artificial neural network according to the measured GPS data and leveling data.

2- Developing an undulation interpolation algorithm using the grid-based geoid model given a point’s plane coordinates, and developing a computer program according this algorithm, then transforming this program to a pocket PC (MATLAB).

3- Estimating the interested point’s undulation in the field using the MATLAB and transforming the GPS height to orthometric height in real-time.

The proposed algorithm and the detailed test results will be presented in this paper.

The coordinate system of GPS is the World Geodetic System of 1984 (WGS-84). Positions determined by GPS receivers are expressed in geocentric coordinate (X, Y, Z) or geodetic coordinate (φ, λ, h) defined by WGS-84 ellipsoid. But in engineering application, those coordinates need to be transformed to local coordinate system (such as Northing and Easting), these two surfaces do not coincide due to the fact that the physical earth is different from the mathematical earth.

The height difference between the geodetic height h and the orthometric height H is called undulation N. (Alevizakou, E. G and Evangelia Lambrou, 2011). The ellipsoidal height, h is measurable with the GPS, while orthometric height, H is observable with levelling operations corrected for gravimetric observations.

The geoid height or geoid undulation, N is the elevation of the geoid surface above the ellipsoid surface at the ground point projected on the Geoid along the geoid vertical (Figure 4).

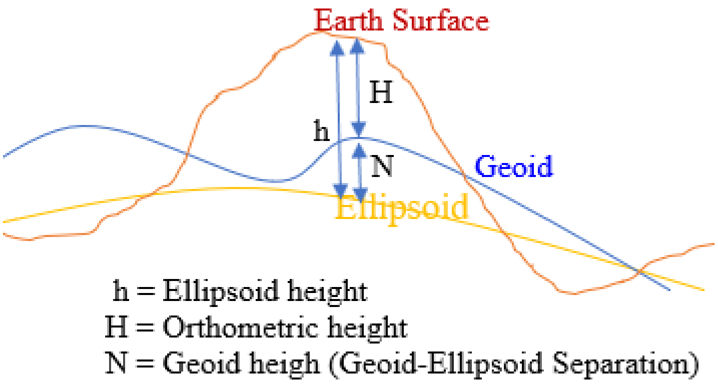

Figure 4: Relation between Earth surface, Geoid and Ellipsoid.

The fundamental relationship between ellipsoidal height (h) obtained from GPS measurements and orthometric heights (H) with respect to a vertical geodetic datum established from spirit levelling data with gravimetric corrections referred to the geoid is given by Heiskanen and Moritz (1969) and Moka and Agajelu (2006) as:

$$N = h - H, \text{ or } H = h - N \qquad (1)$$

Where N =geoid undulation (Geoid separation);

h = ellipsoidal height measured along the ellipsoidal normal, measured by GPS;

H = orthometric height measured along the plumb line, determined by spirit levelling, For the traditional altimetry in cartographic work, MSL is conventionally assigned zero elevation (or level), since the surface of the sea is available from almost everywhere. The MSL is sufficiently determined from tide gauge observations over a long period to filter it from the short-term effects of tide.

From now on, the procedure of transforming the ellipsoidal height h determined by GPS to orthometric height H is defined as GPS height transformation. Based on the foregoing discussion, the key issue of GPS height transformation is how to determine the corresponding undulation N of each point on the earth surface. One of the methods of determining undulation N is called “curve fitting method” or spatial interpolation technique. There are various interpolation methods including: kriging. Spline, Natural neighbor, Inverse distance weighting and Polynomial, to mention some, Paramasivam, C. R. and S. Venkatramanan, 2019).

Suppose there are n reference points with known undulations in one area. Using a polynomial surface to fit these known undulations, then finding the coefficient terms of the polynomial by least square adjustment method. One of the polynomial models frequently used is the second degree polynomial given as:

$$N(x, y) = a + bx + cy + dxy + ex^2 + fy^2 + gx^2y^2 \qquad (2)$$

In which, N (x ‹y) is the height difference between those obtained by LEVEL and GNSS, representing Geoid undulation; x, y are the plane coordinates of a point;

METHODOLOGY

The Test Area

The study area is located in King Saud University opposite the College of Administration (Figure 5). The dimensions of the land are 150m x 50m (7500m^2 or 0.0075km^2). The test area as measured using ArcGIS was found to be 7,638.28 m^2.

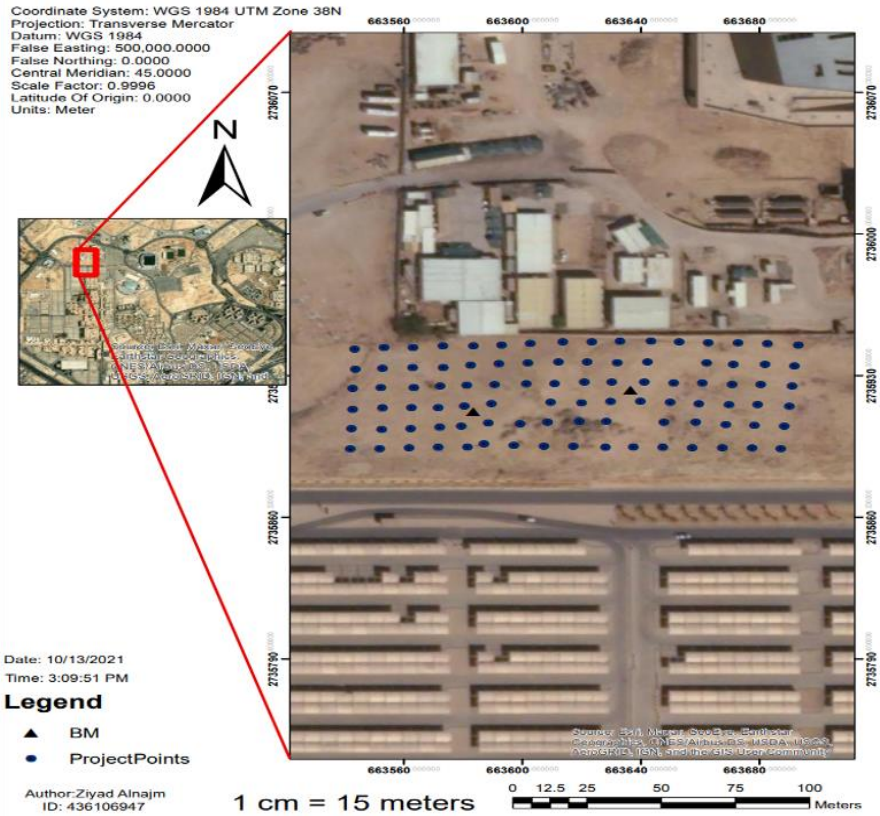

Figure 5: Project location and Test Points.

A total of 90 points regularly distributed with about 10m interval were marked to work as test points. Two points named BM1 and BM2, with coordinates given in Table 1 below were used as control points.

Table 1: Coordinates of Control Points for GPS Observations.

Points	Latitude	Longitude	Height (m)	Stand Deviation in height (m)
BM1	24^{o} 43’ 44.570”N	46^{o} 37’ 02.698”E	660.250	0.012
BM2	24^{o} 43’ 44.893”N	46^{o} 37’ 04.593”E	661.948	0.010

Instrumentation

Leica NA730 automatic level (Figure 6) was available and ready for use in this test. The aim is to determine reduced levels of test points to work as reference for testing elevations of test points produced from GPS observations using Sokkia GRX1 GNSS Instrument (Figure 7).

Figure 6: Leica NA730 (Leica).

Figure 7: Sokkia GRX1 GNSS Instrument and Receiver.

Results of Observations and Data Processing

Contour lines for Orthometric Level Results and GPS Ellipsoidal heights are shown in Fig. 8:

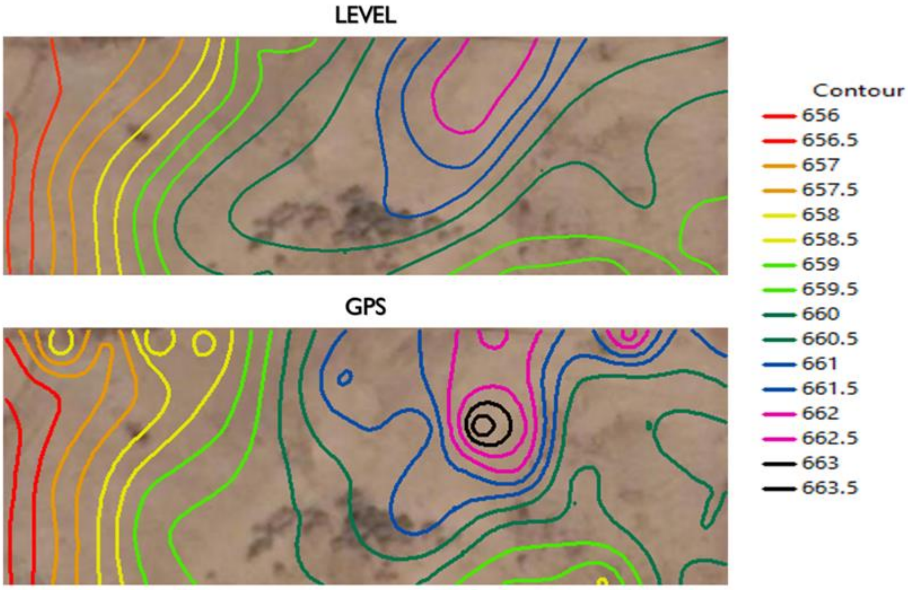

Figure 8: Contour maps for Level (orthometric) and GPS (ellipsoidal heights).

Standard deviation between ellipsoidal and orthometric heights ±0.658m

Polynomial equations used for determining geoid separation:

$$N(x, y) = a + bx + cy + dxy \quad (3)$$

$$N(x, y) = a + bx + cy + dxy + ex^2 + fy^2 \quad (4)$$

RESULTS OF DATA PROCESSING

(i) Using equation (1) and 5 control points as in Table 2:

Table (2) Coordinates and Levels of 5 control points.

Pt. No.	Latitude (m)	Longitude (m)	GPS Height (m)	Level Height (m)	Difference (m)
1	2735942.971	663543.337	656.361	656.295	-0.066
20	2735904.340	663572.023	659.407	659.490	0.083
29	2735936.676	663612.170	661.584	660.510	-1.074
40	2735947.018	663643.427	662.726	662.308	-0.418
60	2735935.412	663671.110	660.110	660.308	0.198

The coefficients a, b, c and d were computed as: a= -27670.125; b=0.08764577; c= 0.00996161; d= -0.000003181. After substituting these coefficients in equation (1) for all points, resulting standard deviation is ±0.309m. Contour lines for Orthometric Level Results and GPS Ellipsoidal heights are shown in Fig. 9:

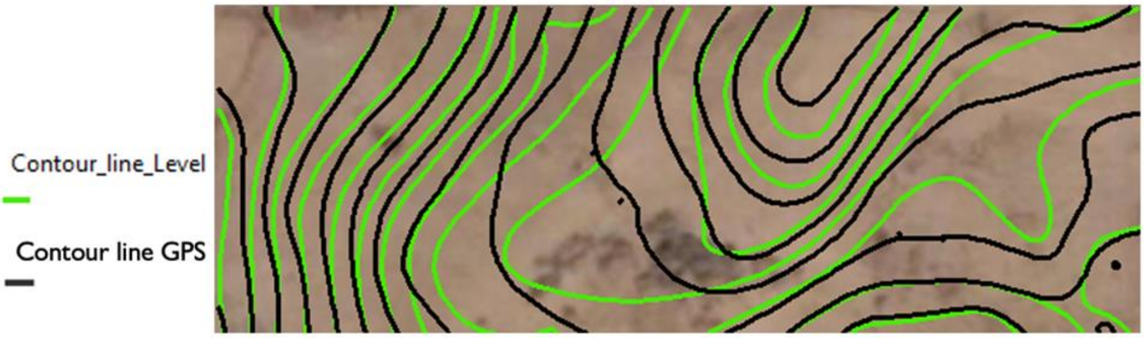

Figure 9: Contour line maps of Level Results and GPS results after determining geoid model using 5 control points and 4 coefficients.

(ii) Using equation (2) with 7 control points (5) points given in Table (3) in addition to two more control points: pt. No.7 and pt. No. 51:

Table (3): coordinates and levels of two points 7 and 51.

Pt. No.	Latitude (m)	Longitude (m)	GPS Height (m)	Level Height (m)	Difference (m)
7	2735893.804	663551.871	657.036	657.055	0.019
51	2735936.124	663662.011	660.419	660.318	-0.101

The computed coefficients a, b, c, d, e and f are:

a = 5975229; b = -7.8579926; c = -2.4523789; d = -0.0000028; e = 0.0000118; f = 0.0000008

Standard error = ±0.375m.

Contour lines for Orthometric Level Results and GPS Ellipsoidal heights are shown in Fig. 10:

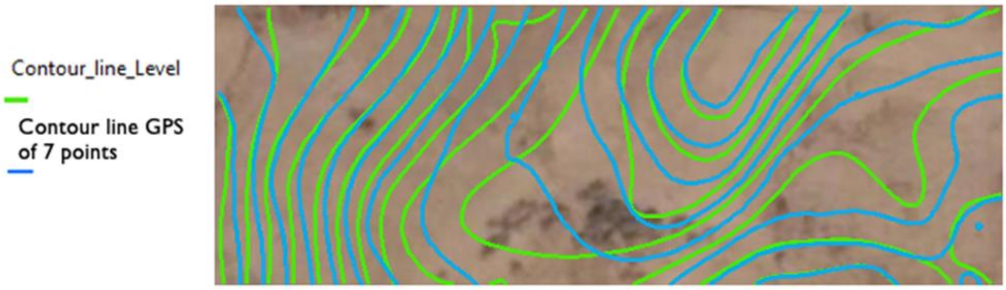

Figure 10: Contour Line maps for Level and GPS after transforming to geoid using equation (2) and 7 control points.

(iii)Using equation (2) with 8 control points: the 7 points in Tables (2) and (3) in addition to point 69 as in Table (4)

Table (4): coordinates and levels of point 69.

Pt. No.	Latitude (m)	Longitude (m)	GPS Height(m)	Level Height(m)	Difference (m)
69	2735914.782	663689.975	660.009	659.868	-0.141

Result of the coefficients a, b, c, d, e and f:

a= 5336622.625; b= -7.3982100; c= -2.0987254

d= -0.0000009; e= 0.00000074; f= 0.0000005

Standard error after checking all points is ±0.229m.

Resulting contour lines for Orthometric Level Results and GPS Ellipsoidal heights are shown in Fig. 11 and Fig. 12:

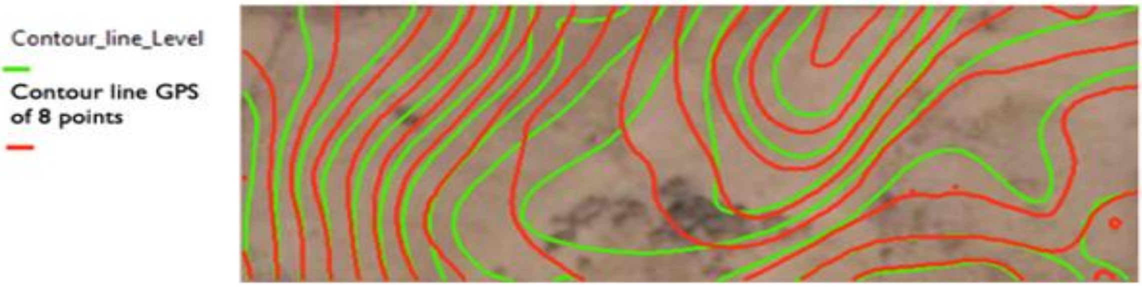

Figure 11: Contour Line maps for Level and GPS after transforming to geoid using equation (2) and 8 control points.

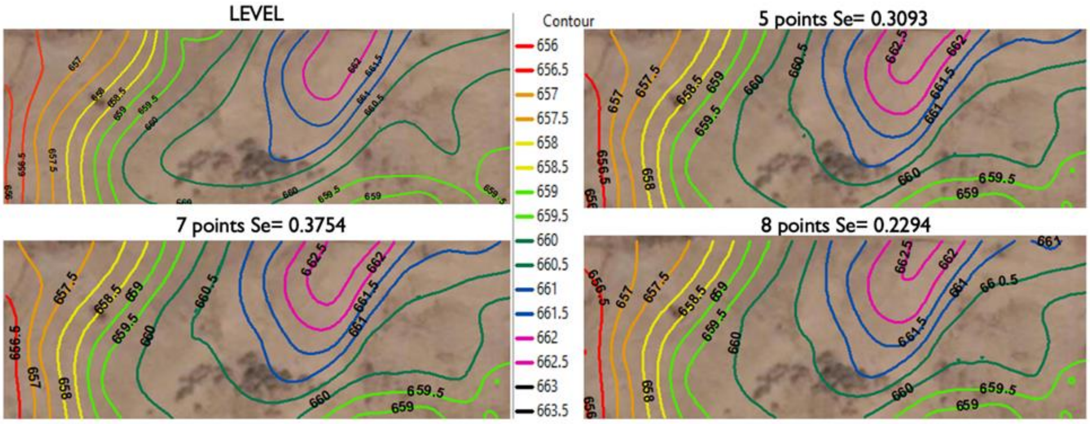

Figure 12: Contour Maps for Level and Orthometric heights obtained from 4 and 6 parameters polynomial geoid models.

CONCLUSIONS

- The study was on a land of 150m x 50m, and this study showed the following
- GPS is the easiest and fastest way, as it saves you time to perform work in the field, and despite the results in this study that with experience, acceptable results will be shown. It also needs some office work before going to the field and needs to correct the points after working in the field from this conclusion, it needs office work before and after the field work, and the results can be presented on the field, but it needs correction.
- The following programs were used in this test: ArcGIS and MATLAB.
- In this study, a 'curve fitting polynomial methods' were applied to relate orthometric height with ellipsoid height and geoid undulation. It has been found that 6 polynomial coefficients with equation, $F(x, y) = a + bx + cy + dxy + ex^2 + fy^2$, with 8 control points showed the best results and the Standard error was ± 0.229m.
- The obtained orthometric heights from GPS observations can be used in various applications including: Hydrologic, Geomorphological, civil engineering earthworks and surveying mapping.

REFERENCES

1. Alevizakou, E. G and Evangelia Lambrou, “Greece Fast and Convenient Geoid Undulation N Determination in an Urban Area”. FIG Working Week 2011 Bridging the Gap between Cultures Marrakech, Morocco, May 2011; 18-22.
2. ARDUSIMPLE, “Ellipsoidal, orthometric and geoid height”. https://www.ardusimple.com/ellipsoidal-orthometric-and-geoid-height/, 2021.

3. Bilani, Hasan and Ismat Elhassan, *"Geoid Modelling for Surveying Works Using Satellite Geo-Spatial Data".* Civil Eng Res J., 2020; 10(2): 24-26. DOI: 10.19080/CERJ. 2020.10.555782.
4. Chun-Jia Huang & Jen-Yu Han "An adaptive approach to optimize regional geoid undulation model for engineering applications", Survey Review, DOI: 10.1080 /00396265. 2022.2096340, 2022.
5. Ellmann A., Vanicek P., UNB application of Stokes's-Helmert's approach to geoid computation. J Geodyn, 2007; 43: 200-213.
6. Erol B. and Rehmi Celik, "Modelling Local Geoid with the assessment of Inverse Distance Weighting and Geostatistical Kriging Methods". ISPRS Commission IV, WG IV/1, 2005.
7. Erol, B., Çelik, R. N., "Precise Local Geoid Determination to Make GPS Technique more Effective in Practical Applications of Geodesy", FIG Working Week, May, Athens, Greece, 2004; 22-27.
8. Eteje S. O., Oduyebo O. F., Olulade Sunday A., "Procedure for the determination of Local Gravimetric-Geometric Geoid Model". International Journal of Advances in Scientific Research and Engineering, IJASRE, 2018; 4(8): 206-214.
9. Eteje S. O., Oduyebo O. F., "Local Geometric Geoid Models Parameters and Accuracy Determination Using Least Squares Technique." International Journal of Innovative Research & Development, July 2018; 7(7). DOI No: 10.24940,
10. Featherstone, W.E. and Dentith, M.C. and Kirby, J.F. Strategies for the accurate determination of orthometric heights from GPS. Survey Review, 1998; 34(267): 278-296.
11. *George D. Garland*, "geoid - The concept of the geoid". Britannica. www.britannica.com/science/geoid/The-concept-of-the-geoid
12. Heiskanen W. A., Moritz H., Physical Geodesy, W H Freeman and Co., San Francisco and London, 1967.
13. Kamguia, J., Charles T. Tabod, R. Nouayou, J.M. Tadjou, E. Manguelle-Dicoum and H.L. Kande, "The Local Geoid Model of Cameroon: CGM05". Nordic Journal, 2007; 4(2).
14. Kaloop, M. R. "Mathematical techniques to compute geoid undulation with GPS data". Conference: The fifth international exhibition and scientific congress "GEO-SIBERIA, 2009.
15. Lars E. Sjöberg, L. E., "On the geoid and orthometric height vs. quasi-geoid and normal height". Journal of Geodetic Science, 2018; 8(1): 115-120. DOI:10.1515/jogs-2018-0011.

16. Leica Automatic Level, www. rise-geo.com/product/surveying-equipment/automatic-levels/leica-na730-auto-level.
17. Lin L. S. 'Orthometric Height Improvement in Tainan City using RTK GPS and Local Geoid Corrector Surface Models. J. Surv. Eng, 2014; 140(1): 35-43.
18. Manisa, M., Rabindra Kumar Das, Mooketsi Segobye and Lopang Maphale, "Developing local geoid model to assess accuracy of orthometric heights from GPS-based ellipsoidal heights in Botswana". Spatial Information Research, 2016; 24(5).
19. Moka, E.C. and Agajelu, S. I., "On the Problem of Computing Orthometric Heights from GPS data". Proceedings of the first international workshop on Geodesy and Geodynamics, 2006; 85-91; November 2006.
20. Mosbeh R. Kaloop, Ahmed Zaki, Hamad Al-Ajami & Mostafa Rabah "Optimizing Local Geoid Undulation Model using GPS/Levelling Measurements and Heuristic Regression Approaches", Survey Review, 2020; 52: 375, 544-554, DOI: 10.1080/00396265. 2019.1665615.
21. Michele Diane Tranes, Thomas H Meyer and Darek Massalski, "Comparisons of GPS-Derived Orthometric Heights Using Local Geometric Geoid Models". Journal of Surveying Engineering, 133(1). DOI:10.1061/(ASCE)0733-9453, 2007; 133: 1(6).
22. Nwilo, P. C. Nwilo, Y. D. Opaluwa, Q. A. Adejare, E. G. Ayodele and A.M. Ayeni. "GEOID MODELLING USING GEOMETRICAL INTERPOLATION METHOD". www.researchgate.net/publication/266731752, 2009.
23. Opa;uwa, Yusuf and Peter Nwile, "Derivation of Orthometric Heights from GPS Measured Heights Using Geometric Technique and EGM96 Model". FUTY Journal of the Environment 02/2011, 2005; 5(1). DOI: 10.4314/fje.v5i1.63477.
24. Pırtı, A., Ramazan Gürsel Hoşbaş and Ho¸sba¸s Ho¸sba¸ Ho¸sba¸s, "Evaluation of some levelling techniques in surveying application". Geodesy and Cartography, 2019; 68(2): 361-373. DOI:10.24425/gac.2019.128463.
25. Paramasivam, C. R. and S.Venkatramanan, "Chapter 3 - An Introduction to Various Spatial Analysis Techniques". GIS and Geostatistical Techniques for Groundwater Science, Elsevier, 2019; 23-30. ISBN 9780128154137.
26. Propeller-Aero, "Geoid vs. Ellipsoid: What is the Surface?" www.propelleraero.com/blog/geoids-vs-ellipsoids-whats-the-difference/, 2021.
27. Rabindra Kumar Das Sailesh Samanta Sujoy Kumar Jana Robert Rosa, "Polynomial interpolation methods in development of local geoid model". The Egyptian Journal of Remote Science and Space Science, 2018; 21(3): 265-271.

28. Sanlioğlu, I., Süleyman Sırrı Maras and Fatih Uysal, “Determination of Orthometric Heights with Real Time Kinematic Surveying, Konya Sample”. FIG Working Week 2009 Surveyors Key Role in Accelerated Development Eilat, Israel, 2009; 3-8.
29. Uzun, S. & Cakir, L. *The Reliability of Surface Fitting Methods in Orthometric Height Determination from GPS Observations. XXIII* FIG Congress, Munich Germany, 2006.
30. Yehia Miky, Reda Yaagoubi and Ahmed Fouad Al Shouny, “Study of the Potential of a Local Geoid Model for Extracting the Orthometric Heights from GPS Measurements in Topographic Works”. Civil and Environmental Research, 2017; 9: 7.
31. Zaki, A., Yasmeen Elberry, Hamad Al-Ajami and Mostafa Rabah, “Determination of local geometric geoid model for Kuwait”. Journal of Applied Geodesy. DOI:10.1515/jag-2022-0017, 2022.
32. Zhan-ji, Yang and Chen Yong-qi, “Determination of Local Geoid Model with Geometric Method: Case Study”. Surveying Engineering Journal, 1999; 125: 3.